MATH 231, FALL 2022—PRACTICE FINAL

1. Solve the initial value problem, including the interval where the solution is defined.

$$xy' - y = x^2 e^x, \quad y = y(x), \quad y(1) = e - 1.$$

2.Solve the initial value problem:

$$(1 + e^x y + xe^x y)dx + (xe^x + 2)dy = 0, \quad y = y(x), \quad y(0) = 1.$$

3. Consider the autonomous first-order equation:

$$y' = -3y(y-1)(y-2), \quad y = y(t).$$

(i) Find and classify the equilibria (as stable or unstable), and sketch the phase-line diagram.

(ii) Sketch the y vs. t graph of solutions (include two solution curves in each range defined by the equilibria.) Indicate the vertical asymptotes ("finite-time blowup") where they occur.

4. Transform the Bernouili-type equation given below to a linear equation (you don't need to solve the transformed equation).

$$y' - 5y = -5xy^3, y = y(x).$$

5. Find the general solution of the Cauchy-Euler equation:

$$(t-2)^2 y''(t) - 7(y-2)y'(t) + 7y(t) = 0, \quad t > 2.$$

6. Solve the initial-value problem. (You may use Laplace transforms):

$$y'' + 6y' + 9y = t, \quad y = y(t), \quad y(0) = 0, y'(0) = 1.$$

7. Find a formula for the solution of the initial-value problem, for any given function $g(t)$:

$$y'' + 2y' + 5y = g(t), \quad y = y(t), \quad y(0) = 2, y'(0) = -2.$$

8. Find the first *three* non-zero terms in a power series expansion of the solution to the first-order initial-value problem:

$$x' + (\sin t)x = 0, \quad x = x(t), \quad x(0) = 1.$$

Given: $\sin t = t - \frac{t^3}{31} + \frac{t^5}{5!} - \frac{x^7}{7!} + \dots$

9. Find power series expansions about $x_0 = 0$ for two linearly independent solutions; include three nonzero terms for each solution:

$$y'' - x^2 y' - xy = 0, \quad y = y(x).$$