MATH 231–SECOND MIDTERM–NOV 10, 2022. (10 pts per problem, 5 pts per item.) Closed book, closed notes, no calculators.

NAME:

1. Find the general solution:

$$(i)y'' - y' + 7y = 0$$

$$(ii)(t-1)^2 y'' - 7(t-1)y' + 7y = 0, \quad y = y(t), t > 1.$$

2. Use undetermined coefficients to find the general solution:

$$y'' + y' = x + \sin 2x, \quad y = y(x).$$

3. Find the general solution (use variation of parameters):

$$y'' + y = f(t) = \sec t, \quad y = y(t), -\pi/2 < t < \pi/2.$$

Given: $\int \tan u\, du = -\ln|\cos u| + C$, and recall:

$$y = v_1 y_1 + v_2 y_2, \quad v_1' = -\frac{y_2 f}{W}, \quad v_2' = \frac{y_1 f}{W},$$

where W is the Wronskian of y_1 and y_2.

4. A 400g mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 10cm (use $g = 10m/s^2$). The damping constant is 3N-sec/m. After the spring comes to rest, at time $t = 0$ an external force $\sin 4t$ N is applied to the system.
(i) Write down the differential equation for the motion.
(ii) Determine the amplitude and *period* of the steady-state motion.

5. (i) Solve the initial-value problem:

$$y'' + 3y' + 2y = 0, y(0) = -1, y'(0) = 3.$$

(ii) Sketch the graph of the solution (for $t \in \mathbb{R}$); if the solution has only one zero and only one critical point, find them.