MATH 561, FALL 2022–PROBLEM SET 1

1. Let X be a locally compact, second countable Hausdorff space. Starting from a countable basis of open sets, remove those whose closure is not compact. Prove that the remaining open sets still form a (countable) basis of X.

2. Let X be a normal space. There exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$, $f(x) > 0$ otherwise, if and only if A is a closed G_δ set in A.

3. Let X be a normal space. There exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$, $f(x) = 1$ for $x \in B$, $0 < f(x) < 1$ otherwise, if and only if A and B are disjoint, closed G_δ subsets of A.