

Problem. The graph of the function $y(x) = x^{4/3}$, regarded as a submanifold of R^2 (of class C^1 , but not C^2), does not have an admissible normal radius.

Solution. The (upward) unit normal at the point (x, y) on the graph is $n = \frac{(-y', 1)}{\sqrt{1+(y')^2}}$. Thus two segments normal to the graph, of length L , drawn from the points (x, y) and $(-x, y)$ on the graph ($x > 0$), will intersect on the y axis if:

$$Ly' = x\sqrt{1+(y')^2},$$

which quickly leads to:

$$L^2 = \left(\frac{x}{y'}\right)^2 + x^2.$$

As $x \rightarrow 0_+$, we see that $L \rightarrow 0$. Thus no admissible radius exists. Since for this function $y(0) = y'(0) = 0$ and $y'(x)$ is monotone, clearly this is a reflection of the fact $y(x)$ is not twice-differentiable at $x = 0$. The same would happen for $y(x) = |x|^\alpha$, for any $1 < \alpha < 2$.