MATH 668 PLANNED OUTLINE (Optimistically; structure follows D. Lee's text):

1. Review of Riemannian geometry/ scalar curvature results

2. First and second variations of volume/ minimizing hypersurfaces and scalar curvature

2A: The Yamabe problem

3. The positive mass theorem (asymptotically flat case): proof via minimizing hypersurfaces

3A: Mass of asymptotically hyperbolic manifolds

4: The Riemannian Penrose Inequality and Inverse Mean Curvature Flow

5: Spin geometry and Witten's proof of the positive mass theorem

5A: Mass, capacity, potential theory approach

6: Quasi-local mass: Brown-York, Wang-Yau and Bartnik's conjecture

7: The EInstein equations: 3+1 decomposition, dominant energy condition, constraint equations

8: The spacetime positive mass theorem and Penrose Conjecture

MATH 668 READING LIST (Spring 2023)

1. **Classical PMT and geometric approaches to the mass**

R. Bartnik; The mass of an asymptotically flat manifold, Comm. Pure Appl. Math (1986)

B. Michel: Geometric Invariance of mass-like asymptotic invariants, ArXiV Dec 2010

M. Herzlich, Computing asymptotic invariants with the Ricci tensor on asymptotically flat and asymptotically hyperbolic manifolds, Annales IHP (2016)

M. Herzlich: Universal Positive Mass Theorems, Comm. Math Phys (2017)

2. **Riemannian Penrose Inequality and Inverse Mean Curvature Flow**

M. Herzlich: A Penrose-like inequality for the mass of Riemannian asymptotically flat manifolds, Comm. Math. Phys, (1997)

G. Huisken, T. Ilmanen: The Inverse Mean Curvature Flow and the Riemannian Penrose Inequality, J. Diff. Geom (2001)

H. Bray: Proof of the Riemannian Penrose Inequality using the positive mass theorem, J. Diff. Geom. (2001)

H. Bray, K. Iga: Superharmonic functions in R^n and the Penrose inequality in General Relativity, Comm. Anal. Geom (2002)

H. Bray, D. Lee: On the Riemannian Penrose inequality in dimensions less than 8, ArXiv (May 2007)

A.Neves: Insufficient Convergence of Inverse Mean Curvature Flow on Asymptotically Hyperbolic Manifolds, ArXiv (Nov 2007)l

M. Mars: Present status of the Penrose Inequality (survey), Class. Quantum Grav. (2009)

O-K Hung, M-T Wang: Inverse Mean Curvature Flows in Hyperbolic 3-space revisited (ArXiv Sep 2014)

3. Quasilocal Mass, Bartnik Conjecture

R. Bartnik: New definition of quasilocal mass, Phys. Rev. Lett. 1989

R. Bartnik: Quasi-spherical metrics and prescribed scalar curvature, J. Diff. Geometry (1993)

Y. Shi, L-F Tam: Positive Mass Theorem and the boundary behavior of compact manifolds with nonnegative scalar curvature, J. Diff. Geometry (2002)

P. Miao: On a localized Riemannian Penrose Inequality, Comm. Math Physics (2009)

P. Miao, Y. Shi, L-F Tam: On Geometric Problems related to Brown-York and Liu-Yau quasilocal mass, ArXiv, Jun 2009

J. Jauregui, Fill-ins of nonnegative scalar curvature, static metrics, and quasilocal mass, Pacific J. Math (2013)

J. Jauregui, P. Miao, L-F Tam: Extensions and Fill-Ins with nonnegative scalar curvature, Class. Quantum Grav. (2013)

Y. Shi, L-F Tam: Positivity of the Brown-York mass with quasi-positive boundary data, ArXiV , Apr 2019

Z. An, L-H Huang: Existence of static vacuum extensions with prescribed Bartnik boundary data,

ArXiv, Jul 2021

4. Positive Energy Theorems for Asymptotically Hyperbolic Manifolds

P.Aviles, R. Mc Owen: Conformal deformation to constant negative scalar curvature on noncompact Riemannian manifolds, J. Diff. Geom (1988)

M. Min-Oo: Scalar Curvature Rigidity of asymptotically hyperbolic spin manifolds, Math. Ann. (1989)

L. Andersson, P. Crusciel, H. Friedrich: On the regularity of solutions to the Yamabe equation and the existence of (…) Comm. Math Phys (1992)

J-P Bourguignon, P. Gauduchon: Spineurs, Operateurs de Dirac et Variations de Metriques, Comm. Math Phys (1992)

L. Andersson, M. Dahl: Scalar Curvature Rigidity for asymptotically locally hyperbolic manifolds, Ann. Global Anal. Geom (1998)

X. Wang: The mass of asymptotically hyperbolic manifolds, J. Differential Geometry (2001)

P. Crusciel, M. Herzlich: The Mass of asymptotically hyperbolic Riemannian manifolds, Pacific J. Math (2003)

P. Crusciel, G. Nagy: The mass of spacelike hypersurfaces in asymptotically anti-de Sitter spacetimes(Hamiltonian approach), ArXiv Jan 2002

L. Andersson, M. Cai, G. Galloway, Rigidity and Positivity of Mass for Asymptotically hyperbolic manifolds, Ann. IHP (2008)

K-K Wong, L-F Tam: Limit of quasilocal mass integrals in asymptotically hyperbolic manifolds, ArXiV Oct 2010

P. Crusciel, E. Delay, Gluing constructions for asymptotically hyperbolic manifolds of constant scalar curvature, Comm. Anal. Geom (2018)

P. Crusciel, G. Galloway, L. Nguyen, T-T Paetz, On the mass aspect function and positive energy theorems for asymptotically hyperbolic manifolds, Class, Quantum Grav (2018)

P. Crusciel, E. Delay: Exotic hyperbolic gluings (of AH initial data sets). J Diff. Geom (2018)

P. Crusciel, E. Delay, The hyperbolic positive energy theorem, ArXiv Jan 2019

L-H Huang, H-C Jang, D. Martin: Mass Rigidity for Hyperbolic Manifolds, ArXiv Apr 2019

P. Crusciel, G. Galloway: Positive Mass Theorems for asymptotically hyperbolic Riemannian manifolds with boundary, ArXiv, July 2021

P. Crusciel, Hyperbolic positive energy theorems, ArXiv, Dec.2021 (survey)

H-C Jang, P. Miao: Hyperbolic Mass via horospheres, ArXiV, Mar 2022

5. Mass, Capacity and Harmonic Functions

R. Schoen, S-Y Yau: Conformally flat manifolds, Kleinian groups and scalar curvature, Invent. Math (1988)

G. Carron, M. Herzlich: The Huber theorem for non-compact conformally flat manifolds, Comm. Math. Helvetici (2002)

A.Freire, F. Schwartz: Mass-Capacity Inequalities for Conformally Flat Manifolds with Boundary, Comm.PDE (2014)

K-K Wong, P. Miao: A new monotone quantity along the Inverse Mean Curvature Flow in R^n, Pacific J. Math (2014)

C. Mantoulidis, P. Miao, L-F Tam: Capacity, quasi-local mass and singular fill-ins, J. Reine u, Angewandte Mathematik, 2020.

H. Bray, D. Kazaras, M Khuri, D. Stern: Harmonic Functions and the Mass of 3-dimensional asymptotically flat Riemannian manifolds, J. Geometric Analysis (2022)

V. Agostiniani, L. Mazzieri,, F. Oronzio: A Green's function proof of the Positive Mass Theorem, ArXiv, Aug 2021

P. Miao: Mass, Capacitary Functions and the Mass-to-Capacity ratio, ArXiv, Oct 2022

5. Spacetime Theorems

M. Herzlich: The positive mass theorem for black holes, revisited, J. Geom. Phys (1998) [Spacetime, spinor proof, with a boundary]

J. Corvino, Scalar curvature deformation and a gluing construction for the Einstein constraint equations, Comm. Math. Phys (2000)

H. Bray. S. Hayward, M. Mars, W. Simon: Generalized Inverse mean curvature flows in spacetime, Comm. Math. Phys (2007)

L. Andersson, J. Metzger: Curvature estimates for stable MOTS, J. Diff. Geom. (2010)

M. Eichmair, L-S Huang, D. Lee, R. Schoen: The spacetime Positive Mass Theorem in Dimensions Less than Eight, Arxiv, Oct 2011

M. Eichmair, The Jang equation reduction of the spacetime positive energy theorem in dimensions less than eight, Comm. Math Phys (2013)

L-S Huang, D. Lee, Rigidity of the Spacetime Positive Mass Theorem, ArXiV Jun 2017

6. Surveys:

J. M Lee, T. Parker: The Yamabe problem, Bulletin AMS (July 1987)

R. Schoen, Variational Theory for the Total Scalar Curvature Functional for Riemannian metrics and related topics (Calc. of Variations, Montecatini 1987, Springer LNM 1365)

R. Bartnik, J. Isenberg: The Constraint Equations, in the Einstein equations and the Large Scale behavior of gravitational fields (P. Crusciel, H. Friedric, eds. (2004)

H. Bray, P. Crusciel, The Penrose Inequality (same volume, 2004)