MATH 562, SPRING 2026- TOPICS

1-topologies on spaces of maps

2-theorems of Arzela-Ascoli and Stone-Weierstrass

3-Baire spaces, generic properties

4-differentiable manifolds and differentiable maps

5-tangent space, differential of a map, tangent bundle, vector bundles

6-IFT, local forms of immersions and submersions/submanifolds and embeddings

7-transversality and regular values

8-paracompactness, partitions of unity, applications (Riem. metrics, embeddings into R^n)

9-semicontinuous functions, Whitney topology on spaces of differentiable maps

10-stability of immersions, embeddings, transversality

11-approximations and smoothing

12-manifolds with boundary, transversality with boundary

13-one-manifolds, nonexistence of retractions, Brouwer fixed-point theorem

14-genericity of transversality (parametric), homotopy transversality theorem

15-Nullsets on manifolds, Sard's theorem

16-Whitney's embedding and immersion theorems

17-mod 2 intersection theory/mod 2 degree and winding number

18-smooth Jordan-Brouwer theorem and Borsuk-Ulam theorem

19-covering automorphisms/properly discontinuous group actions and regular covers

20-existence of simply-connected covering spaces/regular covers and normal subgroups

21-orientable manifolds, oriented double cover, orientation induced by a map/on the bdry

22-oriented intersection theory

23-Brouwer degree and applications; oriented winding number

24-Hopf's theorem on maps from n-dimensional manifolds to the n-sphere

25-Index of vector field singularities and Euler characteristic; Lefschetz fixed point theory

MATH 562, SPRING 2026-SYLLABUS

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Office Hours: Tu+Th, 2:30 to 3:30, or by appointment (email)

Lectures: Tu +Th, 11:20—12:35, Academic Resources Building 400

Grading: based on problem sets and a final exam

Due dates for problem sets: 10 problems by March 5 (before Spring Break), an additional 10 problems by May 5 (last day). 2 problems from each of 10 problem sets. (Individual work.)

Final exam: date TBA (during finals week)

REFERENCES

[Munkres} J. Munkres, Topology-sect 46, 47, 48

[Lima] E.L. Lima, Fundamental Groups and Covering Spaces (AK Peters, 2003)-ch. 4&5

[GP] V. Guillemin, D. Pollack, Differential Topology **(main reference)-**Ch. 1,2,3

[Milnor] J. Milnor, Topology from the Differentiable Viewpoint

[Hirsch] M. Hirsch, Differential Topology (Springer)-ch. 1 to ch. 5

Lecture notes posted on the course page