MATH 661- MODERN TOPOLOGY I-FALL 2024-SYLLABUS

Instructor: Alex Freire (Ayres 325, afreire@utk.edu)
https://web.math.utk.edu/~freire

Lectures: Tu-Th 12:55-14:10, Ayres-G013
Office Hours: Tu 14:30—15:00, or by appointment (email)

Topics course: introduction to classical and recent topics in geometric analysis, with applications to topology of manifolds and general relativity. (Research problems included.)

Recommended: some familiarity with differentiable manifolds, de Rham cohomology and basic concepts of Riemannian geometry (tensors on manifolds, connections, curvature tensor.) Some exposure to linear PDE in Euclidean space would be helpful. (*I'll suggest references to anyone who needs to review this.)*

Main references:

1) The Bochner Technique in Differential Geometry, by Hung-Hsi Wu (2017, Higher Education Press, Beijing)

2) Spin Geometry, by H. Blaine Lawson Jr. and M-L Michelson (1989, Princeton UP)

3) Geometric Relativity, by Dan A. Lee (2019, AMS/GSM 201)

Grading: based on attendance and engagement with the lectures (asking questions) and one 40-min presentation of a topic in the course.

OUTLINE
Hodge theory on compact manifolds

Weitzenböck formulas and applications to topology

Clifford algebras, spin manifolds, spinor bundles and Dirac operators

Introduction to harmonic maps

The Riemannian ADM mass of asymptotically flat and asymptotically hyperbolic manifolds

Harmonic functions on complete manifolds; application to positivity of mass.

Applications to GR: axially symmetric Einstein equations, black hole uniqueness theorems, perfect-fluid matter models.