Problems on finite CW complexes. (Source: [Hatcher], p. 156)

1. If X is a finite CW complex, show that $H_n(X^n)$ is free. (*Hint:* show this group is isomorphic to the kernel of the cellular boundary map:

$$d_n : H_n(X^n, X^{n-1}) \to H_{n-1}(X^{n-1}, X^{n-2}).$$

2. Show the isomorphism between cellular and singular homology is "natural", in the following sense: let $f : X \to Y$ be a *cellular map* (meaning $f(X^n) \subset Y^n$ for all n; X, Y finite CW complexes.) Then f induces a chain map between the cellular chain complexes of X and Y, and the map $f_*^{CW} : H_n^{CW}(X) \to H_n^{CW}(Y)$ induced by this chain map corresponds to $f_* : H_n(X) \to H_n(Y)$ (in singular homology) under the isomorphism $H_n^{CW} \approx H_n(X)$ (singular).

3. If a finite CW complex X is the union of subcomplexes A and B, show that:

$$\chi(X) = \chi(A) + \chi(B) - \chi(A \cap B).$$

4. For $X, \tilde{X}$ finite CW complexes and $p : \tilde{X} \to X$ an n-sheeted covering map, show that $\chi(\tilde{X}) - n\chi(X)$.