MATH 664, SPRING 2026—SYLLABUS

Instructor: Dr. Alex Freire, Ayres 325, afreire@utk.edu

https://web.math.utk.edu/~freire

Office Hours: Tu+Th, 2:30 to 3:30, or by appointment (email)

Lectures: Tu +Th, 12:55—14:10, HSS 59

Grading: based on attendance (w/participation) and written solutions to 10 problems (5 turned in by Spring Break)

COURSE TOPICS AND REFERENCES

PART I: BASIC HOMOTOPY THEORY

The main sources for this part of the course are S-T Hu, *Homotopy Theory* (parts of Ch. I,II,IV,V) and A. Hatcher, *Algebraic Topology* (Ch. 4).

Secondary sources: P.J Hilton, *An Introduction to Homotopy Theory* and A. Fomenko and D. Fuchs, *Homotopical Topology*

TOPICS:

1- Basic problems and constructions: the extension problem (I.1 to I.7), homotopy extension, homotopy equivalence, mapping cylinder, factorization problem, compression, Euclidean neighborhood retracts [1. 8 to 1.12, also Hatcher ch.0]
2- The Hopf theorems and a Hurewicz theorem [Sn,X]<-> Hn(X) if X (n-1)-connected [II.8, II.9]
3- Fibrations, def. via covering homotopy property, fiber bundles [III-2]
4- Hopf fiberings of spheres/ algebraically trivial maps X^3→ S^2 class by H^3(X) [III.5]
5- Lifting and cross-sections, induced fibrations [III 7,8]
6- Homotopy groups, relative htopy groups, boundary operator [IV 2,3,4]
7- Induced homomorphism, algebraic properties, exact sequence [IV 5-7]
8- Homotopy invariance, map induced by fibration [IV 8-10]
9- Role of the basepoint, free homotopies, n-simple spaces [IV 14, 15, 16] Hatcher 4A
10- Htopy groups of product spaces, of wedge of spaces [V.2]
11- Compression Lemma, second Whitehead theorem [Hatcher p. 346]
12- Weak homotopy equivalence of CW complexes (1st Whitehead thm-Hatcher 4.22
13- Hurewicz isomorphism theorem [V.4]
14- Applications: htopy groups of fibrations, of coverings [V 5,6,7]
15- Freudenthal suspension [V.11, see also Hilton VI.2]
16- Hopf invariant [Hilton VI.I], also Hatcher App. 4B

PART II: Fiber bundles, vector bundles, characteristic classes/Obstruction theory

Main reference: The Topology of Fiber Bundles, lecture notes by Ralph L. Cohen (Stanford}, Ch.1,2,3,4.

Secondary source: J. Milnor and J. Stasheff, Characteristic Classes (Princeton U.P)

TOPICS:

17- Vector bundles/ Lie groups and principal bundles/transition functions and structure groups (1.1)
18- Pullbacks and bundle algebra, htopy invariance of fiber bundles (1.2, 2.1)
19- Universal bundles and classifying spaces (2.2)
20- Applications (2.5): line bundles over projective space, structures and htopy liftings, representations and flat connections, embeddings and K-theory
21- Chern classes and Stiefel-Whitney classes (3.2)
22- Product formula and splitting principle (3.3), Pontrjagin classes (3.5)
23- Applications: characteristic classes of manifolds, normal bundles- immersions (3.4)
24- Obstruction theory (4.3)
25- Eilenberg-McLane spaces (4.4)
26- Hopf-Whitney theorem: homotopy construction of cohomology (4.4.2, see also Hatcher 4.3- them 4.57)