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CORROBORATION AND CONDITIONAL POSITIVE RELEVANCE

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In a recent issue of this journal George Schlesinger [1988] has regrettably obscured the subtle elegance of Jonathan Cohen's probabilistic analysis of corroboration. Those acquainted with Cohen's work only through Schlesinger's gloss may accordingly find the following remarks to be of interest.

Suppose that R_1 and R_2 are items of evidence bearing on some proposition S. Perhaps contrary to naive expectation, from

(1) $P(S/R_1) > P(S)$

and

(2) $P(S/R_2) > P(S)$

it follows neither that

(2*C*1) $P(S/R_2R_1) > P(S/R_1)$[1]

nor that

(1*C*2) $P(S/R_1R_2) > P(S/R_2)$.

In short, it may be true of each of two positively relevant items of evidence that neither corroborates the other.

In view of this possibility it is natural to ask what additional conditions will ensure that corroboration of some sort occurs. Jonathan Cohen [1977, pp. 103–107; 1986, p. 512] has furnished an elegant answer to this question, proving that (2C1) is entailed by (1)[2] and (2) supplemented by the additional conditions

(3) $P(R_1R_2) > 0$,

(4) $P(R_2/R_1S) \geqslant P(R_2/S)$,

Philosophical Studies **61**: 295–300, 1991.

(5) $P(R_2/R_1\overline{S}) \leqslant P(R_2/\overline{S})$,

and

(6) $P(S/R_1) < 1$,

and that, with the additional condition

(7) $P(S/R_2) < 1$,

(1C2) follows as well.

In words, if R_1 and R_2 are compatible items of evidence, each positively relevant to S, and R_1 does not establish S with certainty, then, so long as R_1 does not make R_2 less likely when S is true and so long as R_1 does not make R_2 more likely when S is false, we may be sure that R_2 corroborates R_1. If, in addition, R_2 does not establish S with certainty, then we may be sure that R_1 corroborates R_2 as well.

Conditions (4) and (5) are inspired generalizations, respectively, of the conditional independence of R_2 and R_1 given S, and given $\overline{S}$.[3] To the extent that (3)—(6) may be expected to obtain in evidentiary situations where (1) and (2) hold,[4] the naive expectation that R_2 should corroborate R_1 is thus vindicated. To the extent that (7) may be expected to obtain as well, the naive expectation that R_1 and R_2 should be mutually corroborating is similarly vindicated.

One would seemingly be hard pressed to improve on Cohen's analysis, an account of corroboration in probabilistic terms all the more charming for coming from the hand of a scholar known for his profound doubts about representing the weight of evidence as a conditional probability.[5] George Schlesinger [1988] essays this task nevertheless. Schlesinger misconstrues the game at the outset (more on this later) to be one of finding a minimal set of conditions necessary and sufficient for (2C1). He first finds fault with Cohen's conditions on grounds of necessity, but manages to misstate (4) as

(4*s*) $P(R_2/R_1S) > P(R_2/S)$.

and to misstate (5) as

(5*s*) $P(R_2/R_1\overline{S}) > P(R_2/\overline{S})$.[6]

Suppose, however, that Schlesinger had stated (4) and (5) correctly, and had shown, as is easily done,[7] that neither (4) nor (5) is necessary for (2C1). Contrary to Schlesinger's assertion ("He has claimed on a number of occasions, most recently in *Mind*, Oct. 1986, that six premises are required for accomplishing the task," p. 141), Cohen nowhere claims in that article that conditions (1)—(6) are necessary for (2C1).[8] In Cohen [1980, p. 51] it is in fact stated that "R_2 will not *normally* (italics mine) corroborate R_1 at all unless conditions (4) and (5) are satisfied alongside (1), (2), (3), and (6)." Why would he qualify this assertion with the adverb "normally" if he believed that (2C1) entailed (4) and (5) as a simple consequence of the logic of probability?

Indeed, by asking for conditions which, along with (1) and (2), entail (2C1), one precludes at the outset the attainment of a set of conditions necessary for (2C1), for neither (1) nor (2) is necessary for (2C1).[9]

Well, then, perhaps we should drop (1) and (2) as well. Pursuing this line, Schlesinger directs our attention to his own conditions,

(1′) $P(S/R_1) > 0$,

(2′) $P(S/R_2) > 0$,

(ϕ) $P(R_2/SR_1) > P(R_2/\bar{S}R_1)$,

and

(6) $P(S/R_1) < 1$.

Schlesinger offers a correct proof that these conditions are necessary and sufficient for (2C1), although it should be pointed out that (1′), (2′), and (6) are redundant in the presence of (ϕ).[10] But he then makes the false claim ("Obviously in similar fashion we can also prove that $P(S/R_1R_2) > P(S/R_2)$." p. 150) that his conditions entail (1C2) as well.[11]

In Schlesinger's defense it may be protested that he has at least established (modulo our above remarks about the redundancy of three of his conditions) that (ϕ) is an equivalent formulation of (2C1). But it is trivial to generate a score of equivalent formulations of (2C1),[12] which, in standard probabilistic terminology, simply asserts that R_2 is conditionally positively relevant to S, given R_1. The issue is whether any of them, and in particular (ϕ), usefully answers the question of what conditions, along with (1) and (2), entail (2C1). Cohen's satisfying and intuitively appealing answer is that if R_1 and R_2 are each positively relevant to S and the minor boundary conditions (3) and (6) hold, then, so long as R_1 is *not* conditionally negatively relevant to R_2, given S, and R_1 is *not* conditionally positively relevant to R_2, given $\overline{S}$, we may be sure that R_2 is conditionally positively relevant to S, given R_1.

And what does the equivalence of (2C1) and (ϕ), as established by Schlesinger, tell us? It tells us that R_2 is conditionally positively relevant to S, given R_1, if and only if S is conditionally positively relevant to R_2 given R_1, never mind (1) or (2). May we be forgiven for replying, just a bit curtly, that everyone knows (or should know) that positive relevance (conditional or unconditional) is a symmetric relation?

NOTES

[1] In this paper conjunction is indicated by concatenation, and logical negation by a horizontal bar.

[2] As Cohen [1977, 1986] points out, (1) may be replaced with the weaker condition $P(S/R_1) > 0$, although this is more mathematically interesting than it is philosophically interesting, since the philosophical problem in question presupposes that R_1 and R_2 are each positively relevant to S and asks for additional conditions which will entail some sort of corroboration.

[3] The discussion of this point in Cohen [1980] is particularly fine. He first gives a short proof of (2C1) under the stronger conditions of conditional independence represented by the case of equality in (4) and (5), but quickly cautions that this result is unlikely to account in practice for many cases of corroboration, such independence being rather rare. The generalizations of conditional independence represented by (4) and (5), which as Cohen [1986] rightly remarks "are quite tricky to get right," are, on the other hand, just what one would want, since they may be expected to obtain in many (though, of course, not all) cases where (1) and (2) hold.

[4] See note 3 *supra*.

[5] With an admirably stiff upper lip, Cohen [1977, p. 101] titles the relevant section of his book, "A demonstrably adequate analysis of corroboration and convergence in terms of mathematical probability."

[6] Although it hardly matters under the circumstances, the example furnished by Schlesinger to show that (2C1) may be true while (4s) is false is vitiated by an arithmetic error ($P(S) = 5/12$, not $1/6$, as Schlesinger claims) and the example he furnishes to show that (2C1) may be true while (5s) is false is not even a probability.

[7] The following example shows that (2C1) may be true while both (4) and (5) are false:

$$a = P(R_1 R_2 S) = 0.36$$
$$b = P(R_1 \overline{R}_2 S) = 0.16$$
$$c = P(R_1 \overline{R}_2 \overline{S}) = 0.04$$
$$d = P(R_1 R_2 \overline{S}) = 0.08$$
$$e = P(\overline{R}_1 \overline{R}_2 S) = 0.04$$
$$f = P(\overline{R}_1 R_2 S) = 0.16$$

$g = P(\bar{R}_1 R_2 \bar{S}) = 0.04$
$h = P(\bar{R}_1 \bar{R}_2 \bar{S}) = 0.12$

[8] What is of course necessary, if one is to rescue the intuition that positively relevant items of evidence typically corroborate each other, is an analysis *of the type* that Cohen has proffered, with its broadly applicable, intuitively reasonable conditions (4) and (5). Schlesinger has misunderstood Cohen's unexceptionable claim that his analysis partakes of this sort of necessity as the mundane (and false — see note 7 *supra*) claim, never made by Cohen, that (2C1) entails each of the conditions (1)—(6).

[9] The following example, with $a, b, \ldots, h$ as in note 7 *supra*, shows that (2C1) may be true, while both (1) and (2) are false: $a = 0.2$, $b = c = d = 0.1$, $e = 0.25$, $f = g = 0.1$, and $h = 0.05$.

[10] I take it as implicit in the statement

$$(\phi) \qquad P(R_2/SR_1) > P(R_2/\bar{S}R_1)$$

that the conditional probabilities in question are well defined, so that $P(SR_1) > 0$ and thus (since SR_1 entails R_1), $P(R_1) > 0$. Hence $P(S/R_1) = P(SR_1)/P(R_1) > 0$, which is (1′). Moreover, since $P(R_2/\bar{S}R_1) \geqslant 0$, (ϕ) implies that $P(R_2/SR_1) = P(R_2SR_1)/P(SR_1) > 0$, and hence that $P(R_2SR_1) > 0$. Thus (since R_2SR_1 entails both SR_2 and R_2), $P(SR_2) > 0$ and $P(R_2) > 0$, and so $P(S/R_2) = P(SR_2)/P(R_2) > 0$, which is (2′). Finally, the fact that $P(R_2/\bar{S}R_1)$ is well defined implies that $P(\bar{S}R_1) > 0$, and thus that $P(\bar{S}/R_1) > 0$, which implies that $P(S/R_1) < 1$, which is (6).

[11] The following example, with $a, b, \ldots h$ as in note 7 *supra*, shows that (1′), (2′), (ϕ), and (6) may be true, while (1C2) is false: $a = b = 0.1$, $c = 0.2$, $d = e = 0.1$, $f = 0.2$, and $g = h = 0.1$.

[12] Here are seven:

(I, a.k.a. 2C1)	$P(S/R_2R_1) > P(S/R_1)$
(II)	$P(S/R_2R_1) > P(S/\bar{R}_2R_1)$
(III)	$P(R_2/SR_1) > P(R_2/R_1)$
(IV, a.k.a. ϕ)	$P(R_2/SR_1) > P(R_2/\bar{S}R_1)$
(V)	$P(SR_2/R_1) > P(S/R_1)\,P(R_2/R_1)$
(VI)	$P(SR_2/R_1)\,P(\bar{S}\bar{R}_2/R_1) > P(S\bar{R}_2/R_1)P(\bar{S}R_2/R_1)$
(VII)	$P(SR_2R_1)\,P(\bar{S}\bar{R}_2R_1) > P(S\bar{R}_2R_1)\,P(\bar{S}R_2R_1)$

Replacing S by $\bar{S}$ and R_2 by $\bar{R}_2$ in (I)—(V) yields five more conditions to add to the above list. Five more conditions may be generated from (I)—(V) by replacing S by $\bar{S}$ and $>$ by $<$, and five additional conditions by replacing R_2 by $\bar{R}_2$ and $>$ by $<$. Devotees of proving such things by Bayes' Rule should be warned that they are in for a long night. The easier (though still tedious) way to show the equivalence of the aforementioned twenty-two conditions is to express each in terms of the quantities a, b, $c, \ldots h$ defined in note 7 *supra*, and show by elementary algebra that each of the resulting inequalities is equivalent to the simple inequality $ac > bd$, which is precisely condition (VII) above.

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