Hyperbolic functions in case you don't know them

Hyperbolic functions are easily defined in terms of the exponential function:
$\sinh x := (e^x - e^{-x})/2$, $\cosh x := (e^x + e^{-x})/2$, $\tanh x := \sinh x/\cosh x$.

The name similarity with trigs is completely baffling if you look at *graphs* of these functions. Rather, the names are chosen similar to trigs, because *algebraic formulas* for these functions are very similar to those for trig functions.

This similarity from an algebra viewpoint remains a mysterious coincidence as long as you confine yourself to real variable calculus. However, if you study power series (as we will do soon) and allow to plug in complex numbers as well (and we will eagerly do this in defiance of the popular discrimination against complex numbers that is crippling Calculus 2), the similarity between hyperbolic and trig functions will find a simple explanation.

Solving $\sinh x = \frac{e^x - e^{-x}}{2} = t$ for x, we get the inverse hyperbolic function $\operatorname{arsinh} t = \ln(t + \sqrt{t^2+1})$.

Likewise, solving for $x \geq 0$ the equation $\cosh x = \frac{e^x + e^{-x}}{2} = t$, we get the inverse hyperbolic function $\operatorname{arcosh} t = \ln(t + \sqrt{t^2-1})$.

Diatribe: *These hyperbolic functions are* USEFUL *despite being merely abbreviations for simple combinations of exponentials.*

In contrast, I hold the opinion that the invention of the names $\sec x$ *and* $\operatorname{cosec} x$ *for* $1/\cos x$ *and* $1/\sin x$ *respectively is a reckless pollution of the mathematical landscape. In 25 years of math carreer, I haven't had any use for these miscreants; but the stubborn habit of symbolic algebra software like Mathematica to obfuscate the structure of trig expressions by 'simplifying'* $1/\cos x$ *into* $\sec x$ *has given me ample opportunity to lose my temper. The primary sponsors of* sec *and* cosec *are overpriced and obese calculus textbooks that have abdicated their responsibility to teach core material in depth, and instead have padded their pages dealing with these junk functions.*

Another diatribe: *The functions* sinh *and* cosh *are called 'hyperbolic sine and cosine', or by their Latin names 'sinus hyperbolicus' and 'cosinus hyperbolicus'. They are also pronounced 'sinch' (to rhyme with clinch) and 'cosh' (to rhyme with posh) for convenience. Their inverses may be called inverse sinch etc, or by their Latin names 'area (co)sinus hyperbolicus'.*

The inverse trigs are called arcsin *etc because their values can indeed be geometrically represented as the length of an* ARC *on the circle* $x^2 + y^2 = 1$. *They could also represent (twice) the* AREA *of a sector of this circle, so one could instead call them* arsin *(read 'area sine') etc, but nobody chooses this name because the geometric interpretation as an arc seems more natural.*

But the inverse hyperbolic functions deserve to be called arsinh *and* arcosh *and* NOT arcsinh, arccosh, *because they do* NOT *represent* AN ARC *on the hyperbola* $x^2 - y^2 = 1$ *in any natural way, but* ONLY AN AREA *adjacent to this hyperbola. Nevertheless, the whole mathematical community on this continent seems to have conspired to call these functions with an arc-name, thus giving a name analogy precedence over actual meaning.*

Basic Integrals you should know (I omit the C)

$\int x^r\,dx = \frac{x^{r+1}}{r+1}$ for $r \neq -1$ — This includes e.g. $\int \sqrt{x}\,dx$

$\int \frac{dx}{x} = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$ — I prefer this over the $\ln|x|$ variant, because this one protests more loudly if you attempt the felony of integrating across 0. (*)

$\int \sin x\,dx = -\cos x$ and $\int \cos x\,dx = \sin x$

$\int \tan x\,dx = -\ln\cos x$ e.g. in the interval $|x| < \pi/2$ — Shown by substitution $\cos x = u$

$\int \sin^2 x\,dx = \frac{1}{2}(x - \sin x\,\cos x)$ and — Shown either by employing a double-angle trig formula, or by integration by parts

$\int \cos^2 x\,dx = \frac{1}{2}(x + \sin x\,\cos x)$

$\int \frac{dx}{\cos^2 x} = \int (1 + \tan^2 x)\,dx = \tan x$

$\int e^x\,dx = e^x$

$\int \sinh x\,dx = \cosh x$ and $\int \cosh x\,dx = \sinh x$

$\int \tanh x\,dx = \ln\cosh x$ — shown by substitution $\cosh x = u$

$\int \frac{1}{x^2+1}\,dx = \arctan x$

$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1)$ — shown by substitution $u = x^2 + 1$

$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x$

$\int \frac{dx}{\sqrt{x^2+1}} = \operatorname{arsinh} x$ — which actually equals $\ln(x + \sqrt{x^2+1})$

$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{arcosh} x$ — which actually equals $\ln(x + \sqrt{x^2-1})$

$\int \frac{1}{1-x^2}\,dx = \operatorname{artanh} x = \frac{1}{2}\ln\frac{1+x}{1-x}$ — probably the 2nd expression is preferable

$\int \ln x\,dx = x\ln x - x$ — proved by integration by parts with 1 as v'

(*) Moreover, if at some time you allow complex variables x, $\ln|x|$ continues to make sense, but is a wrong antiderivative, whereas the case distinction makes it obvious that this particular formula is not meant for complex variables