UTK – M231 – Differential Equations – Jochen Denzler
Note on Chapter 3 (Modeling)

The trouble with the modeling problems in the book is that they are too standardized. Modeling is never standardized!In a sense the standardized problems are 'too easy', while at the same time they are nevertheless perceived as difficult by many students. So you should not be discouraged by the difficulty. It depends on the timing and class dynamics within the semester whether we can cover this, and so I will determine this at shorter notice. I hold it will be beneficial to most students (the strong ones as well as those who find modeling quite difficult), if we can.

The following modeling problem is intended as a project that extends over several weeks, so that you can come in for discussion and individual feedback repeatedly.

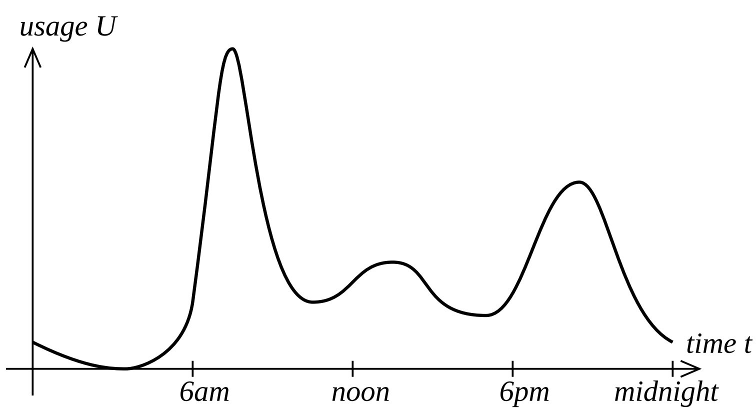
Here is the problem:

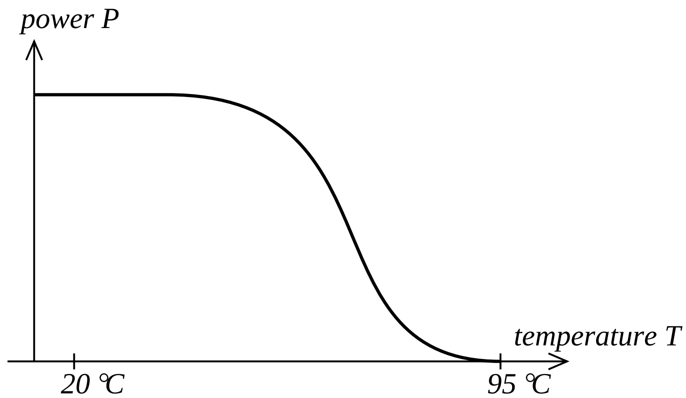
A central hot water system in a house operates with a hot water reservoir of volume $V = 1000L$. If tenants take hot water from the faucet, an equal amount of cold water (temperature $20°C$) from the pipes will replace it. We assume that the water in the container gets mixed fast enough so that the temperature of the water in the container is the same everywhere in the container. The hot water usage U (in L/min) is given as a function of time. We won't specify a formula; a plausible graph may look like the one given below.

The heating system has a temperature sensor that sets the power P (in J/min) automatically in dependence of the temperature T. Again, we don't specify a formula; a plausible P is graphed below.

It takes 4.2×10^3 J to heat one liter of water by $1\ °C$. The physics of heating tells us that the amount of heat needed for heating is proportional to the quantity of water, and also proportional to the temperature difference the heating is to achieve.

Set up the differential equation for the temperature as a function of time.





Hint/Clarification: The functions P and U will enter as 'given' quantities into the ODE; the unknown function in the ODE you try to find will be $T(t)$. You only need to *set up* the ODE. No way of solving it in formulas, since the given functions U and P come as graphs, not as formulas. Moreover the ODE will be non-linear.