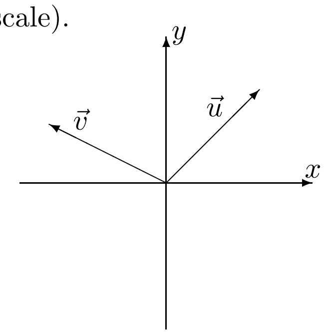
Homework Chapter 3 (being covered before Ch. 2)
UTK – M251 – Matrix Algebra
Fall 2003, Jochen Denzler, MWF 1:25–2:15, Ayres 318

1. Find the components of $\vec{u}$, $\vec{v}$, $\vec{u}+\vec{v}$, and $\vec{u}-\vec{v}$, for the vectors given in the following figure: The radius of the circle is 1; the angles shown are meant to be 45° and 150° (the figure is not quite to scale).



2. Given the point F with coordinates $(0,0,1)$. The following is known about the point P with coordinates (a,b,c): it has the same distance from F as it has from the xy plane. Write this information in form of an equation involving a, b, and c. You may need to think a bit, and reread Ch. 3.2. — FYI: Problem 13 on p. 129 is similar, but not quite the same. And if you need more help, you should come in.

3. Find a unit vector $\vec{x} = [x_1, x_2]^T$ in the plane that is orthogonal to $\vec{w} = [3,4]^T$. How many such unit vectors are there?

4. Find a unit vector $\vec{x} = [x_1, x_2, x_3]^T$ in space that is orthogonal to both $\vec{u} = [1,0,1]^T$ and $\vec{v} = [0,1,1]^T$.

5. Given the vectors $\vec{x} = [x_1, x_2, x_3]^T$ and $\vec{y} = [y_1, y_2, y_3]^T$ in space, I have defined their cross product $\vec{x} \times \vec{y}$ to be the vector $\vec{w} = [x_2y_3 - x_3y_2\,,\ x_3y_1 - x_1y_3\,,\ x_1y_2 - x_2y_1]^T$. Show that indeed, $\|\vec{w}\|^2 = \|\vec{x}\|^2\|\vec{y}\|^2(1-\cos^2\varphi)$, where φ is the angle between $\vec{x}$ and $\vec{y}$.

6. With the same definitions as in the previous problem, check that $\vec{w}$ is orthogonal to $\vec{x}$.

7. Let $\vec{u} = [2,1,3]^T$, $\vec{v} = [-1,0,4]^T$, $\vec{w} = [2,-1,-3]^T$. Calculate $\vec{v}\times\vec{w}$, $\vec{u}\times(\vec{v}\times\vec{w})$, $\vec{u}\times\vec{v}$, $(\vec{u}\times\vec{v})\times\vec{w}$.

8. With the vectors from the previous problem, calculate $\vec{u}\cdot(\vec{v}\times\vec{w})$ and $\vec{v}\cdot(\vec{w}\times\vec{u})$.

9. Find the area of the triangle whose vertices are the points $A(1,1,3)$, $B(-2,3,0)$, $C(1,1,-2)$.

10. Some of the following expressions are meaningless, for vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ in space. Find them, and explain why they are meaningless. For those that *are* meaningful, specify whether the result is scalar or vector.

$$
\begin{array}{ll}
(a) & (\vec{u}\times\vec{v})\times\vec{w} \\
(b) & \vec{u}\times\vec{v}\times\vec{w} \\
(c) & (\vec{u}\cdot\vec{v})\times\vec{w} \\
(d) & (\vec{u}\cdot\vec{v})\vec{w} \\
(e) & (\vec{u}\cdot\vec{v})\cdot\vec{w} \\
(f) & (\vec{u}\times\vec{v})\cdot\vec{w}
\end{array}
$$