

Homework Chapter 7
UTK – M251 – Matrix Algebra
Fall 2003, Jochen Denzler, MWF 1:25–2:15, Ayres 318

The first few problems form a series that requires some calculation, but if you bear with me on them, you get a reasonably *general* picture of eigenvalue problems, rather than encountering special cases that entice you into unwarranted generalizations.

We want to calculate eigenvalues and eigenspaces of the matrix

$$A = \begin{bmatrix} -10 & 28 & -17 & 33 \\ -12 & 40 & -27 & 51 \\ -10 & 35 & -24 & 42 \\ 2 & -7 & 5 & -11 \end{bmatrix}$$

- Without calculation: What degree does the characteristic polynomial of A have?

Now calculate the characteristic polynomial $p(\lambda) = \det(\lambda I - A) = (-1)^4 \det(A - \lambda I)$.

You'll want some hints to check your calculation, because a wrong result here will produce a big mess below: I give away the secret $\det A = -8$. Two checks: read the glossary entry on 'trace and determinant in the characteristic polynomial'

- As $p(\lambda)$ is not quadratic, you need guesswork or numerical methods (built into smart pocket calculator) to find the eigenvalues. Verify that $p(1) = p(-2) = 0$. *If it isn't, you still have a miscalc somewhere.*

Next do a long division of polynomials to find *all* eigenvalues and their algebraic multiplicities.

Check: reread the glossary entry on 'trace and determinant in the characteristic polynomial' and confirm that sum and product of all eigenvalues is what it ought to be.

- Now we want the geometric multiplicities, and bases of all eigenspaces. For each eigenvalue λ , solve the linear system $(A - \lambda I)\mathbf{x} = \mathbf{0}$ and give a basis for the solution space (aka eigenspace for the corresponding eigenvalue).

Make a table:

| λ | alg. mult. | geom. mult. | basis of eigenspace |
|-----------|------------|-------------|---------------------|
| | | | |

Read in page 33–34 of the glossary and mark in the table the piece of information that is responsible that the matrix A is *not* diagonalizable.

- For the matrix $A = \begin{bmatrix} 5 & -9 \\ 3 & -7 \end{bmatrix}$, find matrices P (invertible) and D (diagonal) such that $A = PDP^{-1}$. (There are many correct solutions, but only two possible D can be part of a correct solution.)

- Find an ONB of eigenvectors of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

6. The characteristic polynomial of a certain matrix is $(\lambda - 1)(\lambda + 3)^2(\lambda - 4)^5$.

What is the size of the matrix?

Is the matrix invertible? (Give a reason)

List the eigenvalues and their algebraic multiplicities.

Which possible numbers could be the geometric multiplicity of the largest eigenvalue of this matrix?

7. Assume you have just calculated the eigenvalues of a matrix A , and have come up with the result that $\lambda = \sqrt{2} - 1$ is one of them. Now you look for eigenvectors and after all the calculations you come up with the result that $(A - (\sqrt{2} - 1)I)\mathbf{x} = \mathbf{0}$ has only the solution $\mathbf{0}$. What do you conclude?