Comments on #1:

\( x \in A \cap B \) means: \( x \in A \) and \( x \in B \); y'all know this. BUT

\( x \not\in A \cap B \) doesn't mean "\( x \not\in A \) and \( x \not\in B \)"; and the definition of intersection doesn't justify such a claim.

\( x \not\in A \cap B \) is equivalent to \( x \not\in A \) or \( x \not\in B \).

Sample sol'n: "\( \in \)"

Let \( x \in (A \cap B) \cup (B \cap A) \). This means \( x \in A \cap B \) or \( x \in B \cap A \).

Case 1: \( x \in A \setminus B \), i.e. \( x \in A \), but \( x \notin B \).

Since \( x \in A \), \( x \in A \cap B \) or \( x \in B \), i.e. \( x \in A \cup B \).

We claim that \( x \notin A \cap B \). For if \( x \) were an element of \( A \cap B \), it would in fact be an element of \( B \), contrary to "\( x \notin B \)".

So we have seen \( x \in A \cap B \), but \( x \notin A \cap B \), hence

\( x \in (A \cup B) \setminus (A \cap B) \).

Case 2: \( x \in B \setminus A \), i.e. \( x \in B \), but \( x \notin A \).

(likewise as Case 1, only with \( A, B \) interchanged).

"\( \notin \)" Let \( x \in (A \cup B) \setminus (A \cap B) \). So we have \( x \in A \cup B \), but \( x \notin A \cap B \).

In particular \( x \in A \) or \( x \in B \); but not both, because \( x \notin A \cap B \).

Case 1: \( x \in A \); but then \( x \notin B \).

Hence \( x \in A \setminus B \). Therefore \( x \in (A \setminus B) \cup (B \setminus A) \).

Case 2: \( x \in B \); but then \( x \notin A \).

Hence \( x \in B \setminus A \). Therefore \( x \in (A \setminus B) \cup (B \setminus A) \).

BTW: I phrased the problem "Show that for any sets \( A, B, C \), it holds...".

But \( C \) never occurred in the statement. Technically, it this is