DUE: TUESDAY, JULY 3, 2007 BY 5:00 PM

PROJECT DESCRIPTION:

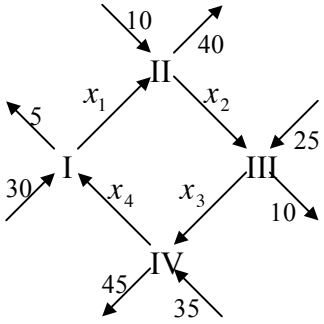
Neatly write up or type up (or a combination of both) in detail the solutions to each of the following exercises, clearly identifying what you are doing in each step. **Points will be deducted if steps are missing, illegible, or difficult to follow.**

I encourage you to use MS Excel (or some other program) and/or your calculator to help you solve each exercise, but you still must write up detailed step-by-step solutions for each problem. I will make available the PIVOT calculator program (see page 285 in the textbook) (available during office hours) and a sample MS Excel spreadsheet (see pages 308-309 in the textbook) (available on the course website at: http://www.math.utk.edu/~dilling/math123/projects/) to assist you.

You may work with others on this project, but each person must hand in his or her own work, and you must **clearly identify every other person you worked with on this project**.

EXERCISES:

1. The following diagram shows the downtown loop of the subway system of a city. The number of subway trains per hour arriving at or leaving stations I, II, III, and IV are indicated by arrows with numbers or variables. (For example, the variables represent how many trains per hour go from one station to the next.) In order for the subway to function, the number of trains arriving and leaving per hour must match at each station. To prevent collisions, the subway trains must travel in the directions indicated by the arrows, and so none of the variables may be negative.



Formulate the scenario as a system of linear equations. Rewrite the system as an augmented matrix and then row-reduce the matrix **making sure to clearly show and describe each step**. (See Example 5 on page 191 in the textbook for help.)

   a. Write the solution set in parameterized form.
   b. For each line (that goes from one station to the next), what is the minimum number of trains per hour that must travel on that line in order to handle the traffic?
   c. The city is thinking about adding a line that runs directly from station I to station III. If 25 trains per hour travel on this line, what is the minimum number of trains per hour that must travel on each of the other lines in order to handle the traffic?

2. A construction foreperson is overseeing a project that will require 5000 work-hours of labor to complete, and she has $85,000 available to pay the workers. Workers are paid $10, $15, or $20 per work-hour depending on their level of experience. Union rules require her to employ as many $20-per-hour workers as $10- and $15-per-hour workers combined.

   Formulate the scenario as a system of linear equations. Be sure to state clearly the meaning of each variable. Rewrite the system as a matrix equation in the form $AX = B$ making sure to write out each matrix explicitly. Answer each of the following questions using the inverse matrix A^{-1} **making sure to clearly show each matrix multiplication** (you may use your calculator to actually do the matrix multiplication) and state your answers in terms of the scenario.

   a. How should she divide up the work-hours between the 3 types of workers?
   b. If the project required 5500 work-hours instead, how should she divide up the work-hours between the 3 types of workers?
   c. If the project required 5000 work-hours but she only had $80,000 available to pay the workers, how should she divide up the work-hours between the 3 types of workers?
   d. Could she complete the project in 5000 work-hours if she only had $70,000 available to pay the workers? (Justify your answer using a mathematical argument.) What if she had $90,000 available?

3. Continuing the scenario from the previous exercise: The construction company earns a profit of $40 per work-hour on $10-per-hour workers, $50 per work-hour on $15-per-hour workers, and $60 per work-hour on $20-per-hour workers. As before, the project must be completed in no more than 5000 work-hours and at a cost of no more than $85,000 for labor. The union rules actually require the foreperson to employ at least as many $20-per-hour workers as $10- and $15-per-hour workers combined.

   Formulate the scenario as a linear programming problem by identifying the variables, the objective function, and the constraints. Be sure to state clearly the meaning of each variable. Check that the problem is a standard maximum problem and then solve it by the simplex method. State your answers in terms of the scenario.

   a. How should the foreperson divide up the work-hours between the 3 types of workers in order to make the most profit for the company? What is the maximum profit?
   b. Interpret the meaning of the final value of each of the slack variables.
   c. How much would the company's profit increase if the maximum number of work-hours available for the project increased from 5000 to 5500?
   d. How much would the company's profit decrease if the maximum amount available to pay workers decreased from $85,000 to $80,000 (assuming a maximum of 5000 work-hours are available for the project)?

GRADING:

35 points for each exercise (105 points total)
5 points per (full or partial) day will be deducted if the project is submitted late.

If you have questions about the project, please let me know.