Name

YOU MUST SHOW WORK FOR EACH PROBLEM IN ORDER TO RECEIVE CREDIT.
SHOW AS MUCH WORK AS POSSIBLE BECAUSE YOU MAY RECEIVE PARTIAL CREDIT FOR THE WORK YOU DO IF YOUR ANSWER IS INCORRECT.

1. 100 bacteria are placed in a Petri dish. After 100 minutes, the population has **grown exponentially** to 1000.

   a. What is the **growth rate** (***r***) of the population?

$$N(t) = N_0 e^{rt}$$
$$N(0) = N_0 = 100$$
$$N(100) = 100e^{100r} = 1000$$
$$e^{100r} = 10 \Rightarrow 100r = \ln 10$$
$$r = \tfrac{\ln 10}{100} \approx 0.023$$

   b. If the population were allowed to continue **growing exponentially** at this rate, **how much longer** would it take for the population size to reach 2000?

$$\text{how much longer} = \text{Doubling time} = \tfrac{\ln 2}{r} \approx \tfrac{\ln 2}{0.023} \approx 30.1 \text{ minutes}$$
$$\text{Check: } N(130.1) \approx 100e^{0.023 \cdot 130.1} \approx 2000$$

2. Once the population size reaches 1000, an antibacterial substance is added to the Petri dish, which causes the population to **die off exponentially**.

   a. If half of the bacteria have died after 50 minutes, what is the **decay rate** (***r***) of the population?

$$N(t) = N_0 e^{-rt}$$
$$N(0) = N_0 = 1000$$
$$N(50) = 1000e^{-50r} = 500$$
$$e^{-50r} = 0.5 \Rightarrow -50r = \ln 0.5 \Rightarrow 50r = \ln 2$$
$$r = \tfrac{\ln 2}{50} \approx 0.014$$

   b. If the population continues to **die off exponentially** at this rate, **how much longer** will it take for the population to return to its initial population size?

$$N(t) = N_0 e^{-rt}$$
$$N(0) = N_0 = 1000$$
$$N(t) = 1000e^{-0.014t} = 100$$
$$e^{-0.014t} = 0.1 \Rightarrow -0.014t = \ln 0.1 \Rightarrow 0.014t = \ln 10$$
$$t = \tfrac{\ln 10}{0.014} \approx 166.1$$
$$\text{how much longer} = 166.1 - 50 = 116.1 \text{ minutes}$$