Name

SHOW AS MUCH WORK AS POSSIBLE BECAUSE YOU MAY RECEIVE PARTIAL CREDIT FOR THE WORK YOU DO IF YOUR ANSWER IS INCORRECT.

For each of the following recursively-defined sequences:

- Starting with $x_0 = 2$, find x_1, x_2, and x_3. (It is okay to use rounded-off decimals.)
- Find the equilibria of the sequence. (There may be zero, one, or more than one equilibria.)
- For each equilibrium, state whether it is stable or unstable (when $x_0 = 2$).

1. $x_{n+1} = \sqrt{4x_n - 3}$

$$\boxed{\begin{aligned} x_1 &= \sqrt{4\cdot 2 - 3} = \sqrt{5} = 2.2361 \\ x_2 &= \sqrt{4\cdot 2.2361 - 3} = 2.4381 \\ x_3 &= \sqrt{4\cdot 2.4381 - 3} = 2.5985 \end{aligned}}$$

$$\boxed{\begin{aligned} & A = \sqrt{4A-3} \Rightarrow A^2 = 4A - 3 \\ & \Rightarrow A^2 - 4A + 3 = 0 \Rightarrow (A-1)(A-3) = 0 \\ & \Rightarrow A = 1 \text{ (unstable)} \\ & \Rightarrow A = 3 \text{ (stable)} \end{aligned}}$$

2. $x_{n+1} = 4x_n - 3$

$$\boxed{\begin{aligned} x_1 &= 4\cdot 2 - 3 = 5 \\ x_2 &= 4\cdot 5 - 3 = 17 \\ x_3 &= 4\cdot 17 - 3 = 65 \end{aligned}}$$

$$\boxed{\begin{aligned} & A = 4A - 3 \Rightarrow -3A = -3 \\ & \Rightarrow A = 1 \text{ (unstable)} \end{aligned}}$$

3. $x_{n+1} = 2x_n - \dfrac{(x_n)^2}{3}$

$$\boxed{\begin{aligned} x_1 &= 2\cdot 2 - \frac{2^2}{3} = 2\tfrac{2}{3} = 2.6667 \\ x_2 &= 2\cdot 2.6667 - \frac{2.6667^2}{3} = 2.9630 \\ x_3 &= 2\cdot 2.9630 - \frac{2.9630^2}{3} = 2.9995 \end{aligned}}$$

$$\boxed{\begin{aligned} & A = 2\cdot A - \frac{A^2}{3} \Rightarrow A^2 - 3A = 0 \Rightarrow A(A-3) = 0 \\ & \Rightarrow A = 0 \text{ (unstable)} \\ & \Rightarrow A = 3 \text{ (stable)} \end{aligned}}$$