A building is infested with cockroaches and an exterminator is called in to handle the problem. The exterminator determines that the cockroach population is doubling (exponentially) every 18 months. At the end of each month, the exterminator will use traps and other techniques that will kill a constant number of cockroaches.

a. By what percentage (rounded to the nearest percent) is the population increasing each month?

$$r = \frac{\ln 2}{t_D} = \frac{\ln 2}{18} = 0.0385$$

$$N(1) = N_0 e^{-0.0385 \cdot 1} = 1.039259 N_0$$

$$\frac{N(1)}{N_0} = \frac{1.039259 N_0}{N_0} \approx 1.04$$

So the population is growing by about 4% each month.

b. Write a difference equation to describe how the population size changes each month that includes its (exponential) growth and the (as yet undetermined) amount that will be killed by the exterminator.

$x_{n+1} = 1.04 x_n - k$ where k is the amount killed by the exterminator each month

c. Since this population is growing (R>1), its equilibrium will be unstable. We know that if the initial population size is below the equilibrium, it will eventually be driven to extinction by the efforts of the exterminator. The exterminator can control the equilibrium value by determining how many cockroaches to kill each month. If there are currently 1000 cockroaches in the population, what is the minimum number the exterminator must kill each month in order to drive the population to extinction?

$$A = \frac{b}{1-a} = \frac{-k}{1-1.04} = \frac{k}{0.04} \Rightarrow k = 0.04A$$

$$A > 1000 \Rightarrow k > 0.04 \cdot 1000 \Rightarrow k > 40$$

So the exterminator must kill more than 40 cockroaches each month to have an equilibrium greater than 1000.

d. If the exterminator kills 200 cockroaches each month, how many months will it take to drive the population to extinction?

$$A = \tfrac{-200}{1-1.04} = 5000$$

$$x_n = (1000 - 5000) \cdot (1.04)^n + 5000$$

$$x_n = -4000 \cdot (1.04)^n + 5000$$

$$0 = -4000 \cdot (1.04)^n + 5000 \Rightarrow -5000 = -4000 \cdot (1.04)^n$$

$$\Rightarrow (1.04)^n = \tfrac{-5000}{-4000} = 1.25 \Rightarrow \ln(1.04)^n = \ln 1.25 \Rightarrow n \ln 1.04 = \ln 1.25$$

$$\Rightarrow n = \frac{\ln 1.25}{\ln 1.04} \approx 5.69$$

So it will take 6 months to drive the population to extinction.

$$x_5 = -4000 \cdot (1.04)^5 + 5000 \approx 133$$

$$x_6 = -4000 \cdot (1.04)^6 + 5000 \approx -61 < 0$$