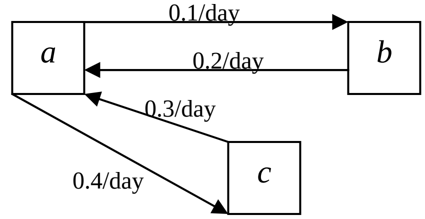
1. The following diagram shows the daily transfer rates of some substance between the 3 compartments of a closed system:



   a. Construct a transfer matrix for this system.

$$T=\begin{bmatrix}0.5 & 0.2 & 0.3\\ 0.1 & 0.8 & 0\\ 0.4 & 0 & 0.7\end{bmatrix}$$

   b. Initially there are 100 grams of the substance in compartment *a*, 200 grams in compartment *b*, and 300 grams in compartment *c*. How many grams of the substance will be in each compartment one day later?

$$\mathbf{x}_1=T\mathbf{x}_0=\begin{bmatrix}0.5 & 0.2 & 0.3\\ 0.1 & 0.8 & 0\\ 0.4 & 0 & 0.7\end{bmatrix}\begin{bmatrix}100\\200\\300\end{bmatrix}=\begin{bmatrix}50+40+90\\10+160+0\\40+0+210\end{bmatrix}=\begin{bmatrix}180\\170\\250\end{bmatrix}$$

   c. How many grams of the substance will there be in the entire system after one year?

The total amount (does not change) = 100 + 200 + 300 = 600

For the following problem, you must show your work in order to receive full credit.

2. For the following Leslie matrix: $\begin{bmatrix}0 & 3\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$

   a. Find all of the eigenvalues and their corresponding normalized eigenvectors.

$$0=\lambda^2-tr(L)\lambda+\det(L)=\lambda^2-\left(0+\tfrac{1}{2}\right)\lambda+\left(0\cdot\tfrac{1}{2}-3\cdot\tfrac{1}{2}\right)$$

$$0=\lambda^2-\tfrac{1}{2}\lambda-\tfrac{3}{2}=\left(\lambda-\tfrac{3}{2}\right)(\lambda+1)$$

$$\lambda=\tfrac{3}{2},\lambda=-1$$

$$L\mathbf{x}=\tfrac{3}{2}\mathbf{x}\Rightarrow\begin{bmatrix}0 & 3\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}j\\a\end{bmatrix}=\tfrac{3}{2}\begin{bmatrix}j\\a\end{bmatrix}\Rightarrow\begin{bmatrix}3a\\ \frac{1}{2}j+\frac{1}{2}a\end{bmatrix}=\begin{bmatrix}\frac{3}{2}j\\ \frac{3}{2}a\end{bmatrix}\Rightarrow 2a=j$$

$$\lambda=\tfrac{3}{2}\Rightarrow\mathbf{x}=\begin{bmatrix}2\\1\end{bmatrix}\div 3=\begin{bmatrix}\frac{2}{3}\\ \frac{1}{3}\end{bmatrix}$$

$$L\mathbf{x}=-\mathbf{x}\Rightarrow\begin{bmatrix}0 & 3\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}j\\a\end{bmatrix}=-\begin{bmatrix}j\\a\end{bmatrix}\Rightarrow\begin{bmatrix}3a\\ \frac{1}{2}j+\frac{1}{2}a\end{bmatrix}=\begin{bmatrix}-j\\-a\end{bmatrix}\Rightarrow -3a=j$$

$$\lambda=-1\Rightarrow\mathbf{x}=\begin{bmatrix}-3\\1\end{bmatrix}\div -2=\begin{bmatrix}\frac{3}{2}\\ -\frac{1}{2}\end{bmatrix}$$

   b. What is the long-term growth rate of the population described by this Leslie matrix?

$$\lambda=\tfrac{3}{2}$$

   c. What is the long-term age distribution of the population? (Assume the population consists of juveniles and adults.)

$$\mathbf{x}=\begin{bmatrix}j\\a\end{bmatrix}=\begin{bmatrix}\frac{2}{3}\\ \frac{1}{3}\end{bmatrix}$$