Name____

Driving back from spring break, I passed a sign that said I was 60 miles from Knoxville. I looked at my speedometer and noticed that I was going exactly 60 mph. For fun, I decided to adjust my speed continuously so that it was always the same as my distance from Knoxville. (ex. when I was 50 miles from Knoxville, I would be traveling at 50 mph, etc.).

From what we learned in section 17, I determined that my velocity could be found using the following function:

$$v(t) = 60e^{-t}$$

where t is the time (in hours) since I passed the sign.

I forgot to look at my odometer when I passed the sign, but I knew that if the function $s(t)$ represented the distance I had traveled at time t since I passed the sign, then my velocity, $v(t)$, must be the derivative of $s(t)$ (i.e. $s'(t) = v(t)$).

1) If my distance from the sign at time $t = 0$ was 0 miles (i.e. $s(0) = 0$), find the function for $s(t)$. *(2 points)*

$$s(t) = -60e^{-t} + C$$

$$s(0) = 0 = -60e^{-0} + C = -60 + C \Rightarrow C = 60$$

$$\boxed{s(t) = 60 - 60e^{-t}}$$

Even though I forgot to keep track of the odometer reading when I passed the sign, I knew I could estimate the distance I would travel in one hour if I kept track of my velocity every 6 minutes (i.e. every 0.1 hours).

2) Use the following table to estimate the distance I traveled in one hour after I passed the sign. *(5 points)*

t	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
$v(t)$	54.3	49.1	44.4	40.2	36.4	32.9	29.8	27.0	24.4	22.1
$v(t)\Delta t$	5.43	4.91	4.44	4.02	3.64	3.29	2.98	2.70	2.44	2.21

$$\int_0^1 v(t)\,dt \approx \sum v(t)\Delta t = \boxed{36.06}$$

I also knew that I could determine the exact distance I had traveled in one hour by using the Fundamental Theorem of Calculus.
(Good thing I remembered this while I was driving! You never know when it will come in handy!)

3) Find $\int_0^1 v(t)\,dt$. *(3 points)*

$$\boxed{\int_0^1 60e^{-t}\,dt = -60e^{-t}\Big|_0^1 = \left(-60e^{-1}\right) - \left(-60e^{0}\right) = 60 - 60e^{-1} \approx 37.93}$$

BONUS: If I had continued to drive in this way, how long would it have taken me to get to Knoxville? *(1 point)*

Forever! $\boxed{\lim_{t\to\infty} s(t) = \lim_{t\to\infty}\left(60 - 60e^{-t}\right) = 60}$