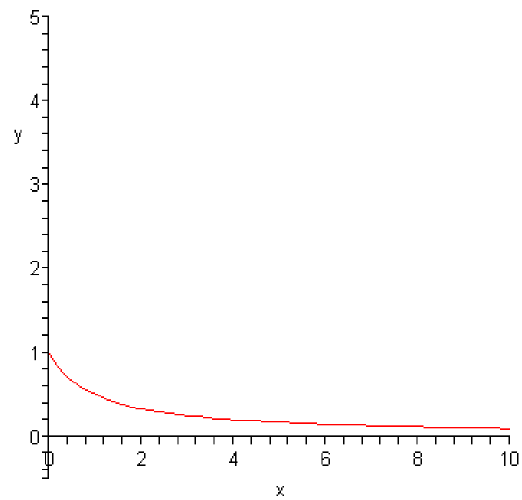
Name___________________________________

HOW MUCH WATER DOES IT TAKE TO FILL A BOTTOMLESS PIT?

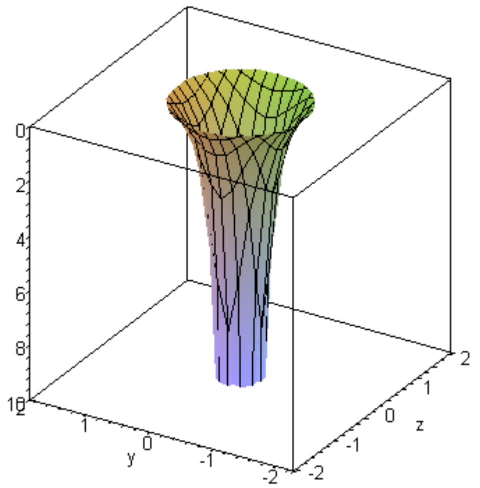
1) Find the area under the curve $y = \frac{1}{x+1}$ for $0 \le x < \infty$. Draw a sketch of the curve first and shade in the area that you are solving for. *(5 points)*

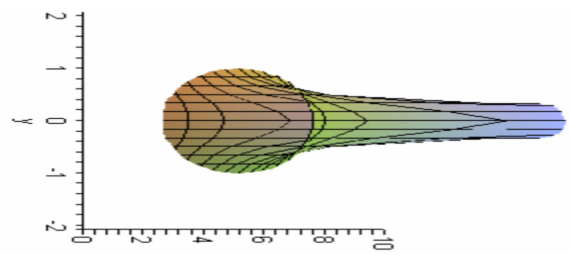


$$\int_0^\infty \frac{1}{x+1}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{x+1}\,dx = \lim_{b\to\infty}\left[\ln(x+1)\right]_0^b = \lim_{b\to\infty}\ln(b+1) - \ln(0+1)$$
$$= \lim_{b\to\infty}\ln(b+1) - \ln 1 = \lim_{b\to\infty}\ln(b+1) - 0$$
$$\boxed{= \lim_{b\to\infty}\ln(b+1) = \infty}$$

So, the area under the curve is **infinite**. (Also, the curve is **infinitely** long).

2) Find the volume of the solid of revolution generated by revolving the curve $y = \dfrac{1}{x+1}$ about the x-axis for $0 \le x < \infty$. Draw a sketch of the solid first. *(5 points)*





$$\int_0^\infty \pi \frac{1}{(x+1)^2}\,dx = \lim_{b\to\infty} \pi \cdot \int_0^b \frac{1}{(x+1)^2}\,dx = \pi \cdot \lim_{b\to\infty}\left[-(x+1)^{-1}\right]_0^b$$

$$= \pi \cdot \left[\lim_{b\to\infty}\left(-\frac{1}{b+1}\right) - \left(-\frac{1}{0+1}\right)\right]$$

$$= \pi \cdot [0 - (-1)] = \pi \cdot 1$$

$$\boxed{= \pi}$$

So, the volume of the solid is **finite** even though it is **infinitely** long (or deep)!