Math 152 – Sample Final Exam – Fall 2006

1. The following function describes the change in temperature during one day in November in Knoxville: $T = 55 + 15\sin\left(\frac{\pi}{12}(t-11)\right)$

(a) What was the maximum temperature for the day and when did it occur?

(b) What was the minimum temperature for the day and when did it occur?

(c) What was the average temperature for the day?

(d) What was the average temperature between noon and 6 PM?

2. Find the derivative of the following functions.

(a) $y = \sqrt{t}\tan\sqrt{t}$

(b) $y = \sec^2 t$

(c) $y = \tan^{-1}\sqrt{t}$

3. The following model describes the fluctuations in a patient's blood pressure over time (in hours): $y'(t) = 10e^{-t}\cos(2t), t \geq 0$ with $y(0) = 120$

(a) When is the patient's blood pressure at a relative maximum or minimum?

(b) When is the patient's blood pressure increasing fastest?

(c) When is the patient's blood pressure decreasing fastest?

(d) As a challenge, solve for $y(t)$.

4. Solve the following integrals.

(a) $\int t^2 \sin t\, dt$ (Hint: Use Integration by Parts twice.)

(b) $\int_0^1 \tan t\, dt$ (Hint: Notice that $\tan t = \frac{\sin t}{\cos t}$ and then use u-substitution.)

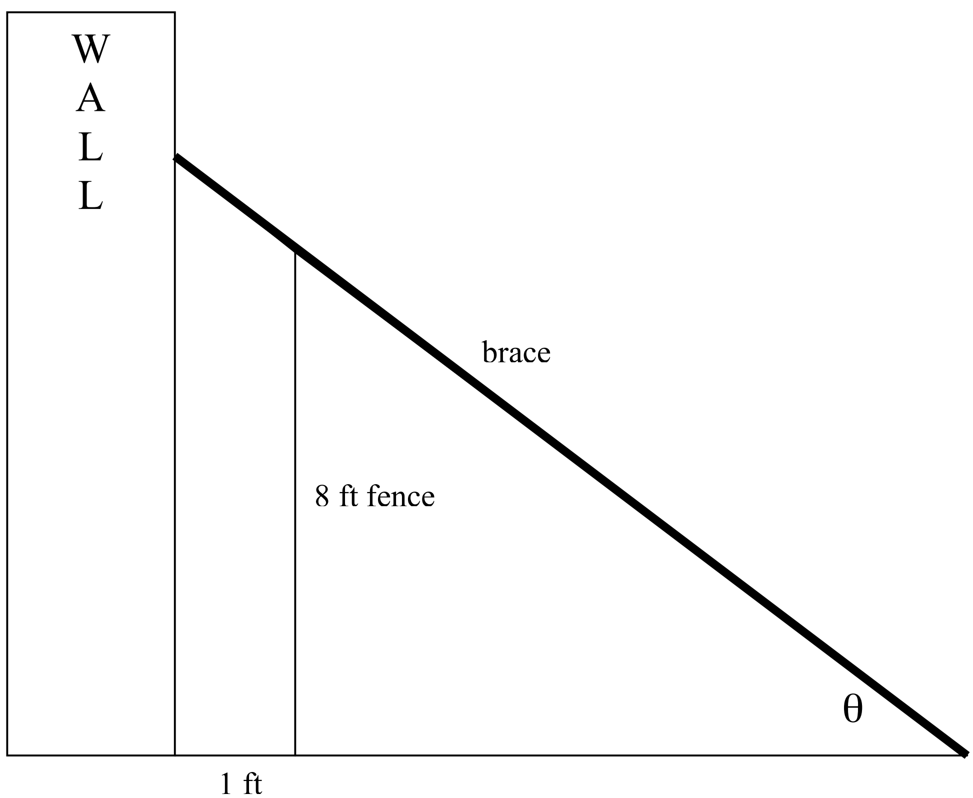
5. Solve the following differential equations subject to the given boundary conditions.

(a) $y' = -\csc y$ with $y(0) = \pi$

(b) $y' = \dfrac{\sin t}{y}$ with $y(0) = 2$

6. A forester is watching a squirrel climbing a tree 30 ft away using binoculars. The squirrel is climbing the tree at a steady rate of 1 foot per second. Find how fast she must increase the angle of her view in terms of the squirrel's height on the tree.

7. A series of braces are being used to reinforce the wall of an animal enclosure. The braces will rest on an 8-foot tall fence that is one foot from the wall of the enclosure. What is the shortest length that each brace can be? (Hint: Express the length of the brace in terms of the angle θ shown in the figure below.)



Math 152 – Sample Final Exam – Answers

1.
(a) 70° at 5 PM
(b) 40° at 5 AM
(c) 55°
(d) 66.7°

2.
(a) $\frac{\tan\sqrt{t}}{2\sqrt{t}} + \frac{\sec^2\sqrt{t}}{2}$
(b) $2\sec^2 t \tan t$
(c) $\frac{1}{2\sqrt{t}(1+t)}$

3.
(a) $t = \frac{\pi}{4} + n\frac{\pi}{2}, n = 0,1,2,\ldots$
(b) At $t = 0$ hours
(c) At $t = \tan^{-1}\left(-\frac{1}{2}\right) + \frac{\pi}{2} \approx 1.1$ hours
(d) $y(t) = 122 - 2e^{-t}\cos(2t) + 4e^{-t}\sin(2t)$

4.
(a) $-t^2\cos t + 2t\sin t + 2\cos t + C$
(b) $-\ln(\cos(1)) = 0.6156$

5.
(a) $y(t) = \cos^{-1}(t-1)$
(b) $y(t) = \sqrt{6 - 2\cos t}$

6. $\frac{d\theta}{dt} = \frac{30}{900 + h^2}$

7. $5\sqrt{5}\, feet = 11.18\, feet$