Math 152 – Sample Exam 3

This exam will cover sections 21, 22, 24, 32, 33, 34, and 35 in the textbook but also relies on your knowledge of derivative and integral rules covered in earlier exams. Look over the earlier exams and make sure you understand how to work the problems on them.

1. Circle City is increasing at the steady rate of 10,000 residents per year. City ordinances require that the population density remain 500 residents per square mile. Consequently, the city continues to expand outward, forming a circle.

(a) How fast is the area of the city growing?

(b) How fast is the radius of the town increasing when the population is 50,000?

2. Find the most general antiderivatives of the following functions.

(a) $f(x) = \sqrt{4x+1}$

(b) $g(t) = \dfrac{t^4}{1+t^5}$

(c) $h(y) = y^2 e^{-y^3}$

(d) $y(x) = xe^{-x}$

(e) $f(x) = \dfrac{2}{x^2 - 2x - 3}$

(f) $f(x) = x\left(x^2+4\right)^8$

3. Find the area under the curve $y = \dfrac{x}{1+x^2}$ from $x = 0$ to $x = 2$.

4. Find the area bounded between the graphs of $y = \sqrt{x}$ and $y = x^4$ for $x \geq 0$.

5. Find the volume of the solid of revolution generated by revolving the curve $y = x^2 + x$ about the x-axis for $0 \leq x \leq 1$.

6. The Michaelis-Menton model describes how an enzyme affects the speed of the chemical transformation of a substrate into a product. The model is very useful for understanding a number of biochemical processes in the body and is described by the following differential equation and boundary condition:

$$\frac{dS}{dt} = -\frac{V_{\max} \cdot S}{S + K_m} \qquad S(0) = S_0$$

where $S(t)$ is the substrate concentration (as a function of time), $V_{\max}$ is the maximum possible speed of the reaction (constant), S_0 is the initial substrate concentration (constant), and K_m is the substrate concentration at which the reaction occurs at the half the maximum possible speed (constant).

(a) Solve the differential equation (and boundary condition) to get an implicit formula for S and t. (You will not be able to solve for S as an explicit function of t.)

(b) For a reaction where $V_{\max} = 5.0 \frac{M}{\text{sec}}$, $S_0 = 0.2M$, and $K_m = 1.0M$, how long after the start of the reaction will it take for the substrate concentration to be reduced to half its initial concentration.

7. Find the following integrals:

(a) $\int_1^2 x \ln x \, dx$

(b) $\int_1^2 \frac{1}{x(x+1)} dx$

(c) $\int_0^1 \frac{x}{\sqrt{2+x}} dx$

Math 152 – Sample Exam 3 – Answers

1.

(a) $\frac{dA}{dt} = 200 \, \frac{mi^2}{year}$

(b) $\frac{dr}{dt} = \frac{\sqrt{\pi}}{\pi} \, \frac{miles}{year} = 0.564 \, \frac{miles}{year}$

2.

(a) $\frac{1}{6}(4x+1)^{3/2} + C$

(b) $\frac{1}{5}\ln\left|1+t^5\right| + C$

(c) $-\frac{1}{3}e^{-y^3} + C$

(d) $-e^{-x}(x+1) + C$

(e) $\frac{1}{2}\ln\left|\frac{x-3}{x+1}\right| + C$

(f) $\frac{1}{18}\left(x^2+4\right)^9 + C$

3. $A = \int_0^2 \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln 5 = 0.805$

4. $A = \int_0^1 \left(\sqrt{x} - x^4\right)dx = \frac{7}{15}$

5. $V = \pi \int_0^1 \left(x^2 + x\right)^2 dx = \frac{31}{30}\pi = 3.246$

6.

(a) $S + K_m \ln|S| = -V_{\max} \cdot t + S_0 + K_m \ln|S_0|$

or $t = \frac{1}{V_{\max}}\left[(S_0 - S) + K_m \ln\left|\frac{S_0}{S}\right|\right]$

(b) $t = \frac{1}{5.0}\left[(0.2-0.1) + 1.0 \cdot \ln\left|\frac{0.2}{0.1}\right|\right] = \frac{1}{5.0}\left[0.1 + \ln 2\right] = 0.159\,\text{sec}$

7.

(a) $2\ln 2 - \frac{3}{4} = 0.636$

(b) $2\ln 2 - \ln 3 = 0.288$

(c) $\frac{8}{3}\sqrt{2} - 2\sqrt{3} = 0.307$