35) $$\frac{1}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}=\frac{A(x+2)+Bx}{x(x+2)}\Rightarrow A(x+2)+Bx=1$$

When $x=0$, $A(0+2)+B\cdot 0=1\Rightarrow A\cdot 2=1\Rightarrow A=\frac{1}{2}$.

When $x=-2$, $A(-2+2)+B\cdot -2=1\Rightarrow B\cdot -2=1\Rightarrow B=\frac{1}{-2}$.

$$\int\frac{1}{x(x+2)}dx=\int\left(\frac{\frac{1}{2}}{x}+\frac{-\frac{1}{2}}{x+2}\right)dx=\frac{1}{2}\int\frac{1}{x}dx-\frac{1}{2}\int\frac{1}{x+2}dx$$

$$=\frac{1}{2}\ln|x|-\frac{1}{2}\ln|x+2|+C=\frac{1}{2}\left(\ln|x|-\ln|x+2|\right)+C=\frac{1}{2}\ln\left|\frac{x}{x+2}\right|+C=\ln\left(\sqrt{\left|\frac{x}{x+2}\right|}\right)+C$$

37) $$\frac{x}{1-x^2}=\frac{x}{(1+x)(1-x)}=\frac{A}{1+x}+\frac{B}{1-x}=\frac{A(1-x)+B(1+x)}{(1+x)(1-x)}\Rightarrow A(1-x)+B(1+x)=x$$

When $x=-1$, $A(1--1)+B(1+-1)=-1\Rightarrow A\cdot 2=-1\Rightarrow A=-\frac{1}{2}$.

When $x=1$, $A(1-1)+B(1+1)=1\Rightarrow B\cdot 2=1\Rightarrow B=\frac{1}{2}$.

$$\int\frac{x}{1-x^2}dx=\int\left(\frac{-\frac{1}{2}}{1+x}+\frac{\frac{1}{2}}{1-x}\right)dx=-\frac{1}{2}\int\frac{1}{x+1}dx-\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=-\frac{1}{2}\ln|x+1|-\frac{1}{2}\ln|x-1|+C=-\frac{1}{2}\left(\ln|x+1|+\ln|x-1|\right)+C$$

$$=-\frac{1}{2}\ln|(x+1)(x-1)|+C=-\frac{1}{2}\ln\left|(x^2-1)\right|+C=-\ln\left|(x^2-1)\right|^{\frac{1}{2}}+C$$

$$=-\ln\left(\sqrt{\left|(x^2-1)\right|}\right)+C=\ln\left(\frac{1}{\sqrt{\left|(x^2-1)\right|}}\right)+C$$

39) $$\frac{x}{(x+2)^2}=\frac{A}{x+2}+\frac{B}{(x+2)^2}=\frac{A(x+2)+B}{(x+2)^2}\Rightarrow A(x+2)+B=x$$

When $x=-2$, $A(-2+2)+B=-2\Rightarrow B=-2$.

When $x=0$, $A(0+2)+B=0\Rightarrow A\cdot 2+-2=0\Rightarrow A=1$.

$$\int\frac{x}{(x+2)^2}dx=\int\left(\frac{1}{x+2}+\frac{-2}{(x+2)^2}\right)dx=\int\frac{1}{x+2}dx-2\int\frac{1}{(x+2)^2}dx$$

$$=\ln|x+2|-2\cdot\frac{1}{-1}\cdot\frac{1}{x+2}+C=\ln|x+2|+\frac{2}{x+2}+C$$

41) $$\frac{3x}{(x+1)^2(x+2)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x+2}=\frac{A(x+1)(x+2)+B(x+2)+C(x+1)^2}{(x+1)^2(x+2)}$$

$\Rightarrow A(x+1)(x+2)+B(x+2)+C(x+1)^2=3x$

When $x=-1$, $A(-1+1)(x+2)+B(-1+2)+C(-1+1)^2=3\cdot -1\Rightarrow B=-3$.

When $x=-2$, $A(-2+1)(-2+2)+B(-2+2)+C(-2+1)^2=3\cdot -2\Rightarrow C=-6$.

When $x=0$, $A(0+1)(0+2)+B(0+2)+C(0+1)^2=3\cdot 0$

$\Rightarrow A\cdot 2+-3\cdot 2+-6\cdot 1=0\Rightarrow A\cdot 2-12=0\Rightarrow A=6$.

$$\int\frac{3x}{(x+1)^2(x+2)}dx=\int\left(\frac{6}{x+1}+\frac{-3}{(x+1)^2}+\frac{-6}{x+2}\right)dx$$

$$=6\int\frac{1}{x+1}dx-3\int\frac{1}{(x+1)^2}dx-6\int\frac{1}{x+2}dx$$

$$=6\ln|x+1|-3\cdot\frac{1}{-1}\cdot\frac{1}{x+1}-6\ln|x+2|+C=6\ln|x+1|+\frac{3}{x+1}-6\ln|x+2|+C$$

$$=6\ln\left|\frac{x+1}{x+2}\right|+\frac{3}{x+1}+C=\ln\left[\left(\frac{x+1}{x+2}\right)^6\right]+\frac{3}{x+1}+C$$

43) $$\frac{x^2}{x^2-1}=\begin{array}{r} 1 \\ x^2-1\overline{)x^2} \\ \underline{x^2-1} \\ 1 \end{array}=1+\frac{1}{x^2-1}=1+\frac{1}{(x+1)(x-1)}=1+\frac{A}{x+1}+\frac{B}{x-1}$$

$$=1+\frac{A(x-1)+B(x+1)}{(x+1)(x-1)}\Rightarrow A(x-1)+B(x+1)=1$$

When $x=-1$, $A(-1-1)+B(-1+1)=1\Rightarrow A\cdot -2=1\Rightarrow A=-\frac{1}{2}$.

When $x=1$, $A(1-1)+B(1+1)=1\Rightarrow B\cdot 2=1\Rightarrow B=\frac{1}{2}$.

$$\int\frac{x^2}{x^2-1}dx=\int\left(1+\frac{-\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}\right)dx=\int 1dx-\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{1}{x-1}dx$$

$$=x-\frac{1}{2}\ln|x+1|+\frac{1}{2}\ln|x-1|+C=x+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C=x+\ln\left(\sqrt{\left|\frac{x-1}{x+1}\right|}\right)+C$$

45) $$\frac{x^3}{x^2-3x+2} = \begin{array}{r|l} & x+3 \\ \cline{2-2} x^2-3x+2 & x^3 \\ & \underline{x^3-3x^2+2x} \\ & 3x^2-2x \\ & \underline{3x^2-9x+6} \\ & 7x-6 \end{array} = x+3+\frac{7x-6}{x^2-3x+2} = x+3+\frac{7x-6}{(x-1)(x-2)}$$

$$= x+3+\frac{A}{x-1}+\frac{B}{x-2} = x+3+\frac{A(x-2)+B(x-1)}{(x-1)(x-2)} \Rightarrow A(x-2)+B(x-1) = 7x-6$$

When $x=1$, $A(1-2)+B(1-1) = 7\cdot 1-6 \Rightarrow A\cdot -1 = 1 \Rightarrow A=-1.$

When $x=2$, $A(2-2)+B(2-1) = 7\cdot 2-6 \Rightarrow B=8.$

$$\int \frac{x^3}{x^2-3x+2}\,dx = \int \left(x+3+\frac{-1}{x-1}+\frac{8}{x-2} \right) dx$$

$$= \int x dx + \int 3dx - \int \frac{1}{x-1}dx + 8\int \frac{1}{x-2}dx$$

$$= \tfrac{1}{2}x^2 + 3x - \ln|x-1| + 8\ln|x-2| + C$$

$$= \tfrac{1}{2}x^2 + 3x + \ln\left|\frac{(x-2)^8}{x-1}\right| + C$$

47) $$\frac{1}{x(ax+b)} = \frac{A}{x}+\frac{B}{ax+b} = \frac{A(ax+b)+Bx}{x(ax+b)} \Rightarrow A(ax+b)+Bx = 1$$

When $x=0$, $A(a\cdot 0+b)+B\cdot 0 = 1 \Rightarrow A\cdot b = 1 \Rightarrow A = \tfrac{1}{b}$.

When $x=-\tfrac{b}{a}$, $A\left(a\cdot -\tfrac{b}{a}+b\right)+B\cdot -\tfrac{b}{a} = 1 \Rightarrow B\cdot -\tfrac{b}{a} = 1 \Rightarrow B = -\tfrac{a}{b}$.

$$\int \frac{1}{x(ax+b)}\,dx = \int \left(\frac{\frac{1}{b}}{x}+\frac{-\frac{a}{b}}{ax+b} \right) dx = \frac{1}{b}\int \frac{1}{x}dx - \frac{1}{b}\int \frac{a}{ax+b}dx$$

$$= \frac{1}{b}\ln|x| - \frac{1}{b}\ln|ax+b| + C = \frac{1}{b}\left(\ln|x| - \ln|ax+b|\right) + C$$

$$= \frac{1}{b}\ln\left|\frac{x}{ax+b}\right| + C = \ln\left(\sqrt[b]{\left|\frac{x}{ax+b}\right|}\right) + C$$