1) Let $w = x$ and $v' = \sin x$.

Then $w' = 1$ and $v = \int \sin x\,dx = -\cos x$.

$\int x\sin x\,dx = x\cdot -\cos x - \int (1\cdot -\cos x)dx = -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

Let $w = x^2$ and $v' = \cos x$.

Then $w' = 2x$ and $v = \int \cos x\,dx = \sin x$.

$\int x^2\cos x\,dx = x^2\cdot\sin x - \int (2x\cdot\sin x)dx = x^2\sin x - 2\int x\sin x\,dx$

$= x^2\sin x - 2(-x\cos x + \sin x) + C = x^2\sin x + 2x\cos x - 2\sin x + C$

3) Let $w = x^3$ and $v' = e^{-2x}$.

Then $w' = 3x^2$ and $v = \int e^{-2x}\,dx = -\tfrac{1}{2}e^{-2x}$.

$\int x^3e^{-2x}\,dx = x^3\cdot -\tfrac{1}{2}e^{-2x} - \int \left(3x^2\cdot -\tfrac{1}{2}e^{-2x}\right)dx = -\tfrac{1}{2}x^3e^{-2x} + \tfrac{3}{2}\int x^2e^{-2x}\,dx$

$= -\tfrac{1}{2}x^3e^{-2x} + \tfrac{3}{2}\left(-\tfrac{1}{2}x^2e^{-2x} + -\tfrac{1}{2}e^{-2x}\left(x - -\tfrac{1}{2}\right)\right) + C$

$= -\tfrac{1}{2}x^3e^{-2x} - \tfrac{3}{4}x^2e^{-2x} - \tfrac{3}{4}xe^{-2x} - \tfrac{3}{8}e^{-2x} + C$

$= -\tfrac{1}{8}e^{-2x}\left(4x^3 + 6x^2 + 6x + 3\right) + C$

5) Let $w = \ln x$ and $v' = x^{-1/2}$.

Then $w' = \dfrac{1}{x}$ and $v = \int x^{-1/2}\,dx = \dfrac{1}{1/2}x^{1/2} = 2x^{1/2}$.

$\int x^{-1/2}\ln x\,dx = \ln x\cdot 2x^{1/2} - \int\left(\dfrac{1}{x}\cdot 2x^{1/2}\right)dx = 2\sqrt{x}\ln x - 2\int x^{-1/2}\,dx$

$= 2\sqrt{x}\ln x - 2\cdot 2x^{1/2} + C = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

7) Let $w = \ln x$ and $v' = 2x + 3$.

Then $w' = \dfrac{1}{x}$ and $v = \int (2x+3)dx = x^2 + 3x$.

$\int (2x+3)\ln x\,dx = \ln x\cdot\left(x^2 + 3x\right) - \int\left(\dfrac{1}{x}\cdot\left(x^2 + 3x\right)\right)dx$

$= \left(x^2 + 3x\right)\ln x - \int (x+3)dx = \left(x^2 + 3x\right)\ln x - \left(\tfrac{1}{2}x^2 + 3x\right) + C$

$= \left(x^2 + 3x\right)\ln x - \tfrac{1}{2}x^2 - 3x + C$

9a) Let $w = x^2$ and $v' = x\sqrt{x^2+1}$.

Then $w' = 2x$ and $v = \int x\left(x^2+1\right)^{1/2}dx = \frac{1}{3}\left(x^2+1\right)^{3/2}$ (by u-substitution).

$$\int x^3\sqrt{x^2+1}\,dx = x^2\cdot\tfrac{1}{3}\left(x^2+1\right)^{3/2} - \int\left(2x\cdot\tfrac{1}{3}\left(x^2+1\right)^{3/2}\right)dx$$

$$= \tfrac{1}{3}x^2\left(x^2+1\right)^{3/2} - \tfrac{2}{3}\int x\left(x^2+1\right)^{3/2}dx$$

$$= \tfrac{1}{3}x^2\left(x^2+1\right)^{3/2} - \tfrac{2}{3}\cdot\tfrac{1}{5}\left(x^2+1\right)^{5/2} + C$$

$$= \tfrac{1}{3}x^2\left(x^2+1\right)^{3/2} - \tfrac{2}{15}\left(x^2+1\right)^{5/2} + C$$

9b) Let $w = x^2$ and $v' = \dfrac{x}{\sqrt{1+x^2}}$.

Then $w' = 2x$ and $v = \int x\left(x^2+1\right)^{-1/2}dx = \left(x^2+1\right)^{1/2}$ (by u-substitution).

$$\int \frac{x^3}{\sqrt{1+x^2}}dx = x^2\cdot\left(1+x^2\right)^{1/2} - \int\left(2x\cdot\left(1+x^2\right)^{1/2}\right)dx$$

$$= x^2\left(1+x^2\right)^{1/2} - 2\int x\left(1+x^2\right)^{1/2}dx$$

$$= x^2\left(1+x^2\right)^{1/2} - 2\cdot\tfrac{1}{3}\left(1+x^2\right)^{3/2} + C$$

$$= x^2\sqrt{1+x^2} - \tfrac{2}{3}\left(1+x^2\right)^{3/2} + C$$

11) Let $u = \sqrt{x}$. Then $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$ and $dx = 2\sqrt{x}\,du = 2u\,du$.

$$\int \sin\sqrt{x}\,dx = \int \sin u\cdot 2u\,du$$

Let $w = 2u$ and $v' = \sin u$.

Then $w' = 2$ and $v = \int \sin u\,du = -\cos u$.

$$\int \sin\sqrt{x}\,dx = \int \sin u\cdot 2u\,du = 2u\cdot -\cos u - \int\left(2\cdot -\cos u\right)du$$

$$= -2u\cos u + 2\int\cos u\,du = -2u\cos u + 2\sin u + C$$

$$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C = 2\left(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right) + C$$

13) Let $u=-x^2$. Then $\frac{du}{dx}=-2x$ and $dx=\frac{du}{-2x}$ and $x^2=-u$.

$\int x^3e^{-x^2}\,dx=\int x^3e^u\cdot\frac{du}{-2x}=-\frac{1}{2}\int x^2e^u\,du=\frac{1}{2}\int ue^u\,du$

Let $w=u$ and $v'=e^u$.

Then $w'=1$ and $v=\int e^u\,du=e^u$.

$\int x^3e^{-x^2}\,dx=\frac{1}{2}\int ue^u\,du=\frac{1}{2}\left(u\cdot e^u-\int\left(1\cdot e^u\right)du\right)$

$=\frac{1}{2}\left(ue^u-\int e^u\,du\right)=\frac{1}{2}\left(ue^u-e^u\right)+C$

$=\frac{1}{2}\left(-x^2e^{-x^2}-e^{-x^2}\right)+C$

15) Let $u=\sqrt{x}$. Then $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ and $dx=2\sqrt{x}du=2udu$ and $\sqrt{x}=u$.

$\int\sqrt{x}e^{\sqrt{x}}\,dx=\int ue^u\cdot 2udu=\int 2u^2e^u\,du$

Let $w=2u^2$ and $v'=e^u$.

Then $w'=4u$ and $v=\int e^u\,du=e^u$.

$\int\sqrt{x}e^{\sqrt{x}}\,dx=\int 2u^2e^u\,du=2u^2\cdot e^u-\int\left(4u\cdot e^u\right)du$

$=2u^2e^u-4\int ue^u\,du=2u^2e^u-4\left(ue^u-e^u\right)+C=2u^2e^u-4ue^u+4e^u+C$

$=2\left(\sqrt{x}\right)^2e^{\sqrt{x}}-4\sqrt{x}e^{\sqrt{x}}+4e^{\sqrt{x}}+C=2xe^{\sqrt{x}}-4\sqrt{x}e^{\sqrt{x}}+4e^{\sqrt{x}}+C$

$=2e^{\sqrt{x}}\left(x-2\sqrt{x}+2\right)+C$

17) Let $u=x^3$. Then $\frac{du}{dx}=3x^2$ and $dx=\frac{du}{3x^2}$ and $x^3=u$.

$\int x^5e^{x^3}\,dx=\int x^5e^u\cdot\frac{du}{3x^2}=\frac{1}{3}\int x^3e^u\,du=\frac{1}{3}\int ue^u\,du$

Let $w=u$ and $v'=e^u$.

Then $w'=1$ and $v=\int e^u\,du=e^u$.

$\int x^5e^{x^3}\,dx=\frac{1}{3}\int ue^u\,du=\frac{1}{3}\left(u\cdot e^u-\int\left(1\cdot e^u\right)du\right)$

$=\frac{1}{3}\left(ue^u-\int e^u\,du\right)=\frac{1}{3}\left(ue^u-e^u\right)+C$

$=\frac{1}{3}\left(x^3e^{x^3}-e^{x^3}\right)+C=\frac{1}{3}e^{x^3}\left(x^3-1\right)+C$