45) Let $u = ax + b$. Then $\frac{du}{dx} = a$ and $dx = \frac{du}{a}$ and $x = \frac{u-b}{a}$.

$$\int \frac{x}{ax+b}dx = \int \frac{\frac{u-b}{a}}{u}\frac{du}{a} = \frac{1}{a^2}\int \frac{u-b}{u}du = \frac{1}{a^2}\int\left(1-\frac{b}{u}\right)du = \frac{1}{a^2}\left[\int 1du - b\int \frac{1}{u}du\right]$$

$$= \frac{1}{a^2}\left[u - b\ln|u|\right] + D = \frac{1}{a^2}\left[ax + b - b\ln|ax+b|\right] + D = \frac{x}{a} + \frac{b}{a^2} - \frac{b}{a^2}\ln|ax+b| + D$$

$$= \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + D + \frac{b}{a^2} = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$$

where $C = D + \frac{b}{a^2}$ since C, D, a, and b are all just constants.

47) Let $u = x^2 + a^2$. Then $\frac{du}{dx} = 2x$ and $dx = \frac{du}{2x}$ and $x^2 = u - a^2$.

$$\int \frac{x^3}{x^2+a^2}dx = \int \frac{x^3}{u}\frac{du}{2x} = \frac{1}{2}\int \frac{x^2}{u}du = \frac{1}{2}\int \frac{u-a^2}{u}du = \frac{1}{2}\int\left(1-\frac{a^2}{u}\right)du$$

$$= \frac{1}{2}\left[\int 1du - a^2\int \frac{1}{u}du\right] = \frac{1}{2}\left[u - a^2\ln|u|\right] + D = \frac{1}{2}\left[x^2 + a^2 - a^2\ln|x^2+a^2|\right] + D$$

$$= \frac{x^2}{2} + \frac{a^2}{2} - \frac{a^2}{2}\ln\left(x^2+a^2\right) + D = \frac{x^2}{2} - \frac{a^2}{2}\ln\left(x^2+a^2\right) + D + \frac{a^2}{2}$$

$$= \frac{x^2}{2} - \frac{a^2}{2}\ln\left(x^2+a^2\right) + C$$

where $C = D + \frac{a^2}{2}$ since C, D, and a are all just constants.

49) Let $w = x^2$ and $v' = x\sqrt{x^2-a^2}$.

Then $w' = 2x$ and $v = \int x\left(x^2-a^2\right)^{1/2}dx = \frac{1}{3}\left(x^2-a^2\right)^{3/2}$ (by u-substitution).

$$\int x^3\sqrt{x^2-a^2}\,dx = x^2\cdot\tfrac{1}{3}\left(x^2-a^2\right)^{3/2} - \int\left(2x\cdot\tfrac{1}{3}\left(x^2-a^2\right)^{3/2}\right)dx$$

$$= \tfrac{1}{3}x^2\left(x^2-a^2\right)^{3/2} - \tfrac{2}{3}\int x\left(x^2-a^2\right)^{3/2}dx$$

$$= \tfrac{1}{3}x^2\left(x^2-a^2\right)^{3/2} - \tfrac{2}{3}\cdot\tfrac{1}{5}\left(x^2-a^2\right)^{5/2} + C \text{ (by u-substitution)}$$

$$\boxed{= \tfrac{1}{3}x^2\left(x^2-a^2\right)^{3/2} - \tfrac{2}{15}\left(x^2-a^2\right)^{5/2} + C}$$

$$= \tfrac{1}{3}x^2\left(x^2-a^2\right)^{3/2} - \tfrac{1}{3}a^2\left(x^2-a^2\right)^{3/2} + \tfrac{1}{3}a^2\left(x^2-a^2\right)^{3/2} - \tfrac{2}{15}\left(x^2-a^2\right)^{5/2} + C$$

$$= \tfrac{1}{3}\left(x^2-a^2\right)\left(x^2-a^2\right)^{3/2} + \tfrac{1}{3}a^2\left(x^2-a^2\right)^{3/2} - \tfrac{2}{15}\left(x^2-a^2\right)^{5/2} + C$$

$$= \tfrac{1}{3}\left(x^2-a^2\right)^{5/2} - \tfrac{2}{15}\left(x^2-a^2\right)^{5/2} + \tfrac{1}{3}a^2\left(x^2-a^2\right)^{3/2} + C$$

$$\boxed{= \tfrac{1}{5}\left(x^2-a^2\right)^{5/2} + \tfrac{1}{3}a^2\left(x^2-a^2\right)^{3/2} + C}$$