September 28th, 2007

Math 251

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Fall 2007

Name: .....

Student ID (last 5 digits): XXX-X.....

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) Let $\mathbf{u} = (1, -1, 2)$ and $\mathbf{v} = (2, 0, 1)$.

(a) Is the angle between $\mathbf{u}$ and $\mathbf{v}$ greater than $\pi/2$, smaller than $\pi/2$ or equal to $\pi/2$? [Remember, the angle between two vectors is always in the interval $[0, \pi]$.]

(b) Compute $\text{proj}_{\mathbf{v}} \mathbf{u}$.

(c) Compute the *length* of the orthogonal component of $\mathbf{u}$ with respect to the direction of $\mathbf{v}$.

2) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, then compute [justifying or showing work!]:

(a) $\begin{vmatrix} a/7 & b/7 & c/7 \\ -2d & -2e & -2f \\ g & h & i \end{vmatrix} =$

(b) $\begin{vmatrix} f & d & e \\ c & a & b \\ i & g & h \end{vmatrix} =$

(c) $\begin{vmatrix} a-2d & b-2e & c-2f \\ g & h & i \\ d+g & e+h & f+i \end{vmatrix} =$

3) Let $A = \begin{bmatrix} 3 & 1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 0 & 1 \\ -3 & 5 & 1 & 1 & 3 \\ 2 & 0 & 1 & 0 & 2 \\ 1 & -1 & 2 & 0 & 4 \end{bmatrix}$.

(a) How many solutions does the *homogeneous* system $A\mathbf{x} = \mathbf{0}$ have? [You do **not** have to find the solutions!!! Just tell me how many and justify.]

(b) Is A^{T} invertible? If so, compute $\det((A^{\mathrm{T}})^{-3})$, if not, justify.

4) Let $A = \begin{bmatrix} -4 & 5 & 2 \\ 1 & -2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$. Solve, if possible, the two systems:

$$A\mathbf{x} = \mathbf{b} \qquad \text{and} \qquad A\mathbf{x} = \mathbf{c}.$$

5) Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & -4 \\ -2 & 5 & 7 \\ -1 & 2 & 4 \end{bmatrix}$. Compute $A \cdot (B^{\mathrm{T}})^{-1}$.

Scratch: