

Math 351

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Fall 2009

Name:

Student ID (last 6 digits): XXX-

MIDTERM 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Use the *Extended Euclidean Algorithm* to write the GCD of 130 and 61 as a linear combination of themselves. *Show the computations explicitly!*

2) [20 points] Give the set of all solutions of the system

$$2x \equiv 3 \pmod{7}$$

$$12x \equiv 5 \pmod{13}$$

3) [20 points] Let a , b , and n be positive integers. Show that if $a^n \mid b^n$, then $a \mid b$.

4) [20 points] Let n and b be positive integers. Show that $(b - 1) \mid n$ if, and only if, the sum of the b -adic digits of n is divisible by $b - 1$.

5) [20 points] Show that if p and $p^2 + 2$ are both prime, then $p = 3$.

Scratch: