Final

M551 – Abstract Algebra

December 7th, 2011

1. Let p be a prime and G be a *non-abelian* group of order p^3. Prove that $G/Z(G) \cong Z_p \times Z_p$ [where $Z(G)$ is the center of G and Z_p is a multiplicative cyclic group of order p].

2. Let G be a finite *simple* group. Show that if p is the *largest* prime dividing $|G|$, then there is no subgroup $H \leq G$ such that $1 < |G : H| < p$.

3. Let R be a PID. Show that every ideal I of R, with $I \neq 0, R$, is a product of finitely many maximal ideals, and that this decomposition is unique up to reordering.

4. Let R be a noetherian commutative ring with 1 [and $1 \neq 0$] and D be a multiplicative closed subset of R with $1 \in R$ and $0 \notin R$. Let $R_D \stackrel{\text{def}}{=} D^{-1}R$ be the localization of R at D. Show that R_D is also noetherian.