October 5, 2012

Math 251

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Fall 2012

Name: .....

Student ID (last 6 digits): XXX- .....

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	40	
3	20	
4	25	
Total	100	

1) [15 points] Assuming that the system $A\mathbf{x} = \mathbf{b}$ has the reduced row echelon form of its augmented matrix as below, find the solution(s) of the system (or stated that there is no solution if that is the case). No need to show work.

(a) $[A|\mathbf{b}] \sim \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 3 & 0 & 5 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(b) $[A|\mathbf{b}] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

(c) $[A|\mathbf{b}] \sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

2) [40 points] Quickies! (You should be able to answer these quickly and with no, or very little, calculations!) No need to show work!

(a) $\begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 3 & -1 & 0 & 0 \\ 5 & 2 & -3 & 3 & 0 \\ 5 & 0 & 0 & -2 & 5 \end{vmatrix} =$

(b) $\begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 4 \\ 2 & 4 & 2 & 2 \\ -1 & 2 & 4 & 5 \end{vmatrix} =$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} =$

(d) If A is an $n \times n$ matrix with $\det(A) \neq 0$, then how many solutions can the system $A\mathbf{x} = \mathbf{b}$ possibly have?

(e) If A is an $n \times n$ matrix which is not invertible, then how many solutions can the *homogeneous* system $A\mathbf{x} = \mathbf{0}$ possibly have?

(f) $\left(\begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix} - 3 \cdot \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \right)^{\mathrm{T}} =$

(g) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} =$

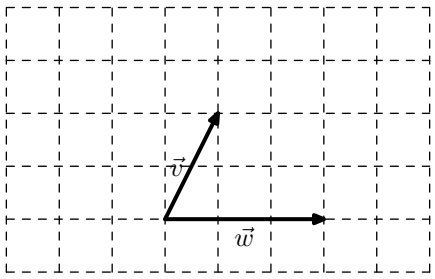
(h) Find the $(2,1)$-entry of the *inverse* of the matrix A below, knowing that $\det(A) = -14$. [**Hint:** You do not need to compute the whole inverse! This is quick if you use cofactors.]

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & -1 \\ 4 & 0 & 2 \end{bmatrix}$$

(i) If A and B are given below, what is the $(3,1)$ entry of $A \cdot B$?

$$A = \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 3 & -1 \end{bmatrix}$$

(j) If $\vec{v}$ and $\vec{w}$ are the vectors given below, draw the vectors $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$ on the grid. [Label each one appropriately!]



3) [20 points] Let $\mathbf{v} = (1, -1, 3)$ and $\mathbf{w} = (2, 2, 1)$. Compute:

(a) $\|\mathbf{v}\|$ [length of $\mathbf{v}$]

(b) $\mathbf{v} \cdot \mathbf{w}$ [dot product]

(c) the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$

(d) the orthogonal projection of $\mathbf{v}$ on $\mathbf{w}$

(e) the component of $\mathbf{v}$ orthogonal to $\mathbf{w}$

4) [25 points] Compute the reduced row echelon form, inverse (if it exits – if not say so and explain why) and determinant of the matrix A below. [**Hint:** You can compute all of them *together*, while finding the reduced row echelon form of A.]

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

Scratch: