November 7, 2012

Math 251

Luís Finotti
Fall 2012

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	15	
3	15	
4	15	
5	15	
6	20	
Total	100	

1) [20 points] Fill in the blanks [no need to justify]:

(a) $\dim(\mathbb{R}^5) =$

(b) $\dim(P_7) =$

(c) $\dim(M_{3\times 2}) =$

(d) If $\text{rank}(A) = 4$ and the system $A\mathbf{x} = \mathbf{b}$ is consistent, then $\text{rank}([A|\mathbf{b}]) =$

(e) If A is an 3×4 matrix for which T_A [the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$] is one-to-one, then $\text{rank}(A) =$

(f) If $T : \mathbb{R}^7 \to \mathbb{R}^5$ is an onto linear transformation, then $\text{rank}([T]) =$

(g) If A is a 4×6 matrix with $\text{nulltiy}(A) = 3$, then:

$$\begin{aligned} \text{dim. of row sp of } A &= \\ \text{dim. of col. sp of } A &= \\ \text{rank}(A) &= \\ \text{rank}(A^{\mathrm{T}}) &= \\ \text{nulltiy}(A^{\mathrm{T}}) &= \end{aligned}$$

2) [15 points] Let $\mathbf{v}_1 = (1, -1, 0, 3)$ and $\mathbf{v}_2 = (1, 0, -1, 0)$. Is $\mathbf{v} = (-1, -2, 3, 6)$ a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$? If so, write $\mathbf{v}$ as such linear combination. [Show work!]

3) [15 points] Let $W = \text{span}\{(1,0,2,1),(0,1,1,1)\}$. Find a basis for the orthogonal complement $W^{\perp}$.

4) [15 points] Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation for which $T(\mathbf{x})$ is given by:

(i) Rotate $\mathbf{x}$ by 45 degrees [counter-clockwise];

(ii) Reflect the resulting vector about the y-axis;

(iii) Project this last vector onto the x-axis.

Find $[T]$ and $T(-2,1)$.

5) [15 points] Let B be the standard basis of P_2 and $B' = \{1, 1+x, 1+x+x^2\}$. You may assume [without proving] that B' is also a basis of P_2.

(a) Find the transition matrix $P_{B \to B'}$.

(b) Find $(1 - 2x + 3x^2)_{B'}$.

6) Let $S = \{(1,0,1,2,1),(0,1,1,-1,2),(-1,2,1,-4,3),(2,1,2,-1,1),(0,0,1,4,3)\}$ and let $V = \text{span}(S)$ [the subspace of $\mathbb{R}^5$ spanned by the set S]. Given that

$$\begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & -1 & 2 \\ -1 & 2 & 1 & -4 & 3 \\ 2 & 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 4 & 3 \end{bmatrix} \xrightarrow{\text{red. ech. form}} \begin{bmatrix} 1 & 0 & 0 & -2 & -2 \\ 0 & 1 & 0 & -5 & -1 \\ 0 & 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 1 \\ 2 & -1 & -4 & -1 & 4 \\ 1 & 2 & 3 & 1 & 3 \end{bmatrix} \xrightarrow{\text{red. ech. form}} \begin{bmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

answer the following. [No need to justify.]

(a) [5 points] Find a basis of V made of vectors in S.

(b) [5 points] If B is the basis you've found in part (a), express the vectors in S that are not in B as a linear combination of vectors in B.

(c) [5 points] Find a second basis B' for V [with $B \neq B'$].

(d) [5 points] Find the coordinates of the first vector of B with respect to B'.

Scratch: