March 14th, 2005

Math 151

Luís Finotti
Winter Quarter 2005

Name: . . . .

SSN (last 4 digit): XXX-XX-. . . . .

Check your Recitation Class:

Yusuf Danisman:	1:30 ☐	2:30 ☐
Bo Dong:	1:30 ☐	2:30 ☐
Jie Wang:	1:30 ☐	2:30 ☐

Final Exam

You have 108 minutes to answer the following 7 questions. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name, the last four digits of your social security number on the top of this page and check the box for your recitation class. Check also that no pages of your exam are missing. This exam has 12 printed and numbered pages.

No books or notes are allowed on this exam! Calculators are permitted only as stated in the "Calculator Policy" of the course.

Show all work! Even correct answers without work may result in point deductions.

Plan your time wisely! It is a good idea to skim over all of the questions before you start. Do not spend too much time on one particular problem.

Good Luck!

Question	Max. Points	Score
1	20	
2	24	
3	40	
4	40	
5	24	
6	24	
7	28	
Total	200	

1. (a) Consider the functions $h(x) = \dfrac{2x}{x-1}$ and $g(w) = \dfrac{w}{w-2}$.

Find the **simplest possible** expression in x for the composite function

$g \circ h(x) = g(h(x)) =$

(b) Suppose for some angle θ between 0 and $\frac{\pi}{2}$ we know that $\cos(\theta) = 1/3$.

Determine the exact expression for $\tan(\theta)$.

2. Determine the following limits: **You must show all work!**

(a) $\lim_{z \to 1^+} \dfrac{1 - \sqrt{z}}{z^2 + z - 2}$ [*Hint:* Extend with conjugate]

(b) $\lim_{\theta \to 0} \dfrac{\tan(2\theta)}{\sin(5\theta)}$

(c) $\lim_{x \to \infty} \dfrac{1 - 3x + 5x^2 - 2x^3}{5x^3 + x - 8}$

3. Find the following:

Show all work!
Do **not** simplify algebraic expressions!

(a) Find the derivative $\dfrac{d}{dt}(t^{21}\tan(3t))$

(b) For $p(\phi) = \dfrac{\cos(\phi) + \phi}{\sqrt[3]{\phi} - \phi^3}$ find $p'(\phi)$.

(c) Suppose $f(x) = (g(x))^3 + g(\sqrt{x})$, and $g(2) = 3$, $g'(2) = 4$, $g(4) = \sqrt{5}$, $g'(4) = \pi$.
Find $f(4)$ and $f'(4)$.

(d) Find the following **antiderivative**:

Show all work!
Do **not** simplify algebraic expressions!

$$\int 2x^2 + \frac{1}{\sqrt{x}} - 2\sin(x)\; dx$$

4. You are given the following function, together with its first and second derivative:

$$f(x) = \frac{x-1}{(x+2)^2} \qquad f'(x) = \frac{4-x}{(x+2)^3} \qquad f''(x) = \frac{2x-14}{(x+2)^4}$$

Show all work! Clearly indicate your answers!

No sketch is required.[1]

Note: All intervals must be contained in the (natural) domain of $f(x)$.

(a) Find the intervals on which $f(x)$ is positive and the intervals on which $f(x)$ is negative.

(b) Find any horizontal asymptotes. Write DNE if none exist. For each case state the respective limits of $f(x)$ that imply the asymptote.

(c) Find any vertical asymptotes. Write DNE if none exist. For each case state the respective limits of $f(x)$ that imply the asymptote.

(d) Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Include all finite endpoints in the intervals as long as they belong to the domain of $f(x)$.

[1] You may use the grid provided on the last page of this exam to check whether your answers are consistent with a sketch of the graph. However, credit will only be given for your written answers here, and **not** for any parts of your sketch.

(e) Find all critical points of $f(x)$ in its domain.

For each critical point decide whether is a local maximum, local minimum or neither.

Explain why!

Determine the values of $f(x)$ at the critical points.

(f) Find the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

Include all finite endpoints in the intervals as long as they belong to the domain of $f(x)$.

(g) Find all inflection points (including x and y coordinates).

5. The curve $\mathcal{C}$ in the xy-plane is given by the equation:

$$y^2 + xy + y = x^3$$

(a) Check that the point $(0, -1)$ lies on $\mathcal{C}$.

(b) Find the equation of the tangent line to the curve $\mathcal{C}$ at $(0, -1)$.

(c) Find a point at which the tangent to $\mathcal{C}$ is horizontal.

[*Hint:* You don't have to find *all* points, only *one* point. Also, here *both* the equation for $\mathcal{C}$ and $\frac{dy}{dx} = 0$ must be fulfilled.]

6. Consider the function $f(x) = 3x^{\frac{2}{3}} + 2$ defined for all real numbers. [2]

For each of the intervals below decide whether there is a point inside interval at which the instantaneous rate of change coincides with the average rate of change over the interval. (That is, a point for which the conclusion of the *Mean Value Theorem* holds.)

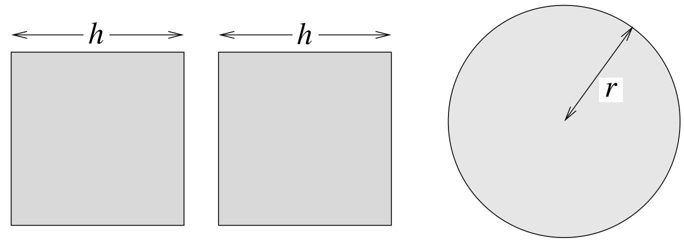
If it does find the point were this occurs. If not give the reason why the conclusion of the *Mean Value Theorem* fails to hold.

(a) $[8, 27]$.

(b) $[-1, 1]$.

[2]The power expression is meant to be the even function $x^{\frac{2}{3}} = \sqrt[3]{x^2}$.

7. A builder wants to fence in three areas. One has the shape of a circle and the other two have the shape of identical squares. (See picture on the right.)



Assume she is using exactly $160 ft$ of fence. Determine the smallest and the largest total area that can be fenced in this way.

Proceed as follows. Show all work and give explanations!

(a) Express the total (combined) area A of the fenced in areas in the variables h and r, given by the width of the squares and the radius of the circle respectively.

(b) Similarly, find an expression in h and r for total length ($= 160 ft$) enclosing the areas, and write down the constraint that is implied by the above conditions.

(c) Determine from this A as a function of only one variable.

Find also the domain of A considering which variables make sense in the given context.

(d) Determine the maximum and the minimum of A on its domain.

Provide all of the relevant arguments and tests.

End !

Additional Workspace:

Warning:

THIS PAGE WILL NOT BE GRADED!

ALL ANSWERS MUST BE GIVEN ON THE PREVIOUS PAGES!