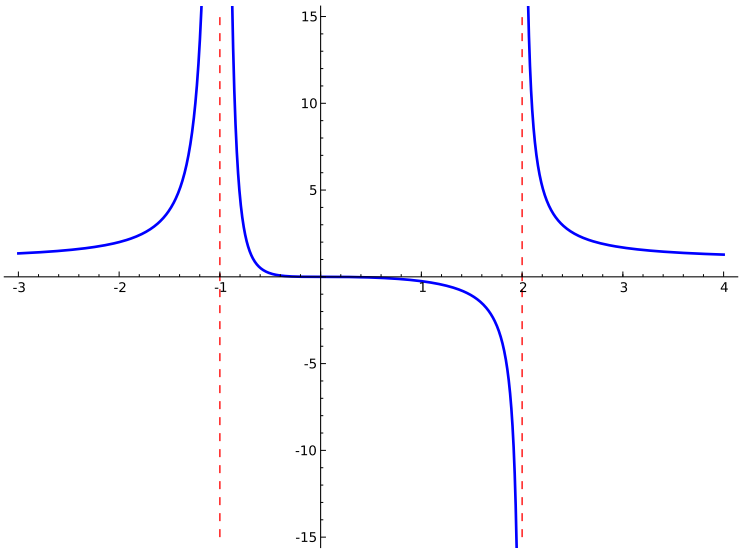
INFINITE LIMIT WORKSHEET

MATH 141 – PROF. L. FINOTTI

Questions

1) Consider the graph of $f(x)$ given below and compute the limits:



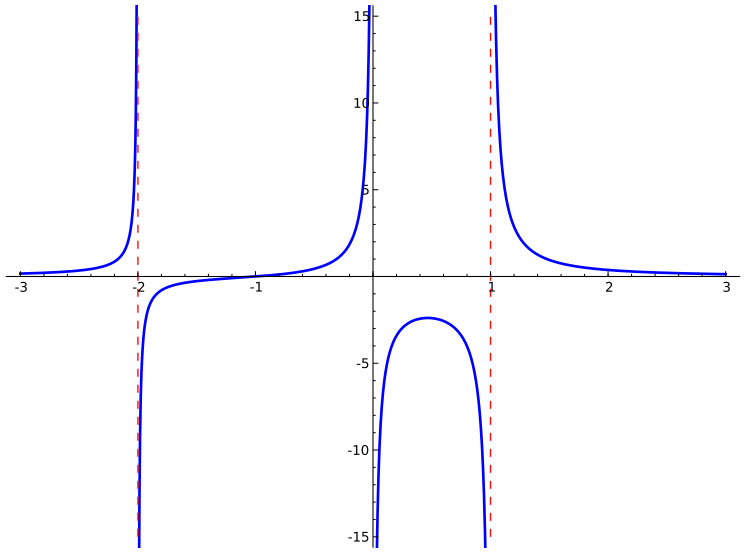
(a) $\lim_{x \to -1^-} f(x) =$

(b) $\lim_{x \to -1^+} f(x) =$

(c) $\lim_{x \to 2^-} f(x) =$

(d) $\lim_{x \to 2^+} f(x) =$

2) Consider the graph of $f(x)$ given below:



At what values of x does $f(x)$ has an infinite limit [as x approaches this value]? Write down the side limits [both!] for each one of these values.

3) Compute the following limits:

(a) $\displaystyle\lim_{x\to 2^{\pm}} \frac{x+1}{x-2}$ [The "$\pm$" here means that you have to compute *both* side limits individually.]

(b) $\displaystyle\lim_{x\to 3^{\pm}} \frac{x^2-3x-1}{x-3}$

(c) $\displaystyle\lim_{x\to 0^{\pm}} \frac{\cos(x^2-x)}{x^2}$

(d) $\displaystyle\lim_{x\to 2^{\pm}} \frac{\mathrm{e}^{x-2}}{x-2}$

(e) $\displaystyle\lim_{x\to 1^{\pm}} \frac{x^2-4x+3}{x^2-2x+1}$

(f) $\displaystyle\lim_{x\to \pi^{\pm}} \frac{\cos(2x)}{\sin(x)}$

(g) $\lim_{x\to\infty} \ln\left(\frac{x^2+1}{x^4}\right)$

(h) $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sin(x)}$

(i) $\lim_{x\to 0^\pm} \frac{\sin(x)}{1-\cos(x)}$

Answers (and Hints) on the next page.

Answers (and Hints)

1: (a): ∞, (b): ∞, (c): $-\infty$, (d): ∞.

2: We have infinite limits for $x = -2, 0, 1$. The side limits are:

$$\lim_{x\to -2^-} f(x) = \infty; \qquad \lim_{x\to -2^+} f(x) = -\infty; \qquad \lim_{x\to 0^-} f(x) = \infty;$$

$$\lim_{x\to 0^+} f(x) = -\infty; \qquad \lim_{x\to 1^-} f(x) = -\infty; \qquad \lim_{x\to 1^+} f(x) = \infty.$$

3: We right first the limit from the left and the from the right in the answers.

(a) $-\infty$, ∞

(b) ∞, $-\infty$

(c) ∞, ∞

(d) $-\infty$, ∞

(e) ∞, $-\infty$. **Hint:** $\dfrac{x^2-4x+3}{x^2-2x+1} = \dfrac{(x-1)(x-3)}{(x-1)^2}$.

(f) ∞, $-\infty$

(g) $-\infty$. **Hint:** $\lim\limits_{x\to 0^+} \ln(x) = -\infty$.

(h) ∞. **Hint:** $\dfrac{\sqrt{x}}{\sin(x)} = \dfrac{\sqrt{x}/x}{\sin(x)/x} = \dfrac{1/\sqrt{x}}{\sin(x)/x}$.

(i) $-\infty$, ∞, **Hint:** $\dfrac{\sin(x)}{1-\cos(x)} = \dfrac{\sin(x)/x}{(1-\cos(x))/x}$.