September 21th, 2016

Math 307

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Fall 2016

Name: .....

Student ID (last 6 digits): XXX- .....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
Total	100	

1) Let $I = \{-1, 0, 1, 2\}$ and $A_i = \{0, i, i^2, -i^3\}$.

(a) [5 points] Write out explicitly the sets A_i for $i \in I$.

(b) [5 points] Find $\bigcup_{i \in I} A_i$.

(c) [5 points] Find $\bigcap_{i \in I} A_i$.

2) [15 points] Express $\exists! x \in A\ (P(x))$ without using $\exists!$.

3) [15 points] Show that

$$\exists x \left(P(x) \to Q(x)\right) \sim \left[\forall x(P(x))\right] \to \left[\exists x(Q(x))\right].$$

4) Analyze the following statements:

(a) [10 points] Someone likes a person who doesn't like anyone.

(b) [10 points] No father is happy unless all his children are happy.

5) Let $A = \{x \in \mathbb{R} \mid x > 0\}$ and consider the statement

$$\neg\left[\forall\epsilon \in A\left(\exists\delta \in A\left(\forall x(|x-a| < \delta \to |x^n - L| < \epsilon)\right)\right)\right].$$

[**Note:** Although irrelevant for your solution, but simply for your information, the statement above is the negation of $\lim\limits_{x\to a} x^n = L$.]

(a) [5 points] State which are the free and which are the bound variables of the statement.

(b) [15 points] Reexpress the statement as its equivalent positive statement.

6) [15 points] Analyze the logical form of the following statement. [You may use $\in$, $\notin$, $=$, $\neq$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\forall$ and $\exists$, but not $\subseteq$, $\nsubseteq$, $\mathscr{P}$, $\cap$, $\cup$, $\setminus$, $\{$, $\}$ or $\neg$.]

$$\bigcap_{i \in I} A_i \subseteq \mathscr{P}(B) \setminus \bigcup_{i \in I} C_i.$$

Scratch: