December 7th, 2016

Math 307

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Fall 2016

Name: ....

Student ID (last 6 digits): XXX- ....

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 11 printed pages (including this one and two pages for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	12	
2	12	
3	12	
4	12	
5	13	
6	13	
7	13	
8	13	
Total	100	

1) [12 points] Let $\mathcal{F}$ and $\mathcal{G}$ be families of sets. Prove that

$$\left(\bigcup \mathcal{F}\right) \setminus \left(\bigcup \mathcal{G}\right) \subseteq \bigcup (\mathcal{F} \setminus \mathcal{G}).$$

Partial credit: If you can't do this or are stuck, I will give half credit [6 points] for the definitions of $x \in X \setminus Y$, $x \in \bigcup \mathcal{F}$ and $\neg(x \in \bigcup \mathcal{G})$.

2) [12 points] Suppose R is a partial order on A, $B_1 \subseteq A$, $B_2 \subseteq A$, x_1 the least upper bound of B_1, and x_2 the least upper bound of B_2. Prove that if $B_1 \subseteq B_2$, then $x_1 R x_2$ [or $x_1 \preccurlyeq x_2$, as I usually write for ordering relations].

[**Hint:** Prove that x_2 is an upper bound of B_1.]

Partial credit: If you can't do this or are stuck, I will give half credit [6 points] for the definitions of upper bound and least upper bound.

[This was a homework problem.]

3) [12 points] Let R be an equivalence relation on a set A. Prove that $[x] \subseteq [y]$ iff xRy. [Remember that $[a]$ denotes the equivalence class of a.]
[This was done in class.]
Partial credit: If you can't do this or are stuck, I will give half credit [6 points] for the definitions of equivalence relation and equivalence class.

4) [12 points] Let $f : A \to C$ and $g : B \to C$. Prove that if A and B are disjoint, then $(f \cup g) : A \cup B \to C$.

[This was a homework problem.]

Partial credit: If you can't do this or are stuck, I will give some credit [4 points] for the definition of a function.

5) [13 points] Let $f : A \to B$ and $g : B \to C$. Prove that if $g \circ f$ is onto, then g is onto.
[This was done in class.]

Partial credit: If you can't do this or are stuck, I will give some credit [4 points] for the definition of an onto function.

6) [13 points] Prove that for any $n \in \mathbb{Z}_{\geq 1}$ we have

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

7) [13 points] Prove that for any $n \in \mathbb{Z}_{\geq 0}$ we have $(n+4)! \geq 4^n$.

8) [13 points] Consider the sequence $a_0, a_1, a_2, \ldots$ recursively defined as follows:

$$a_0 = 1,$$
$$a_{n+1} = 1 + \frac{1}{a_n}, \text{ for } n \geq 0.$$

Prove that for all $n \geq 0$ we have

$$a_n = \frac{F_{n+2}}{F_{n+1}},$$

where $F_0, F_1, F_2, \ldots$ is the Fibonacci sequence.

[**Remember:** $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.]

[This was done in a video.]

Scratch:

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