October 13th, 2017

Math 351

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Fall 2017

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No calculators, books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Find all integers x such that

$$\begin{aligned} 5x &\equiv 3 \pmod{6} \\ 3x &\equiv 1 \pmod{10}. \end{aligned}$$

[If there is no such integer, explain how you could tell.]

2) [20 points] Let

$$n = 604239 \cdot (450027)^{6695} + 7082819.$$

Find its residue modulo 11 [i.e., the remainder when n is divided by 11].

[**Hint:** Remember that if the [decimal] digits of a are given by $a = d_k d_{k-1} \cdots d_0$, then $a \equiv d_0 - d_1 + d_2 - d_3 + \cdots + (-1)^{k-1} d_k + (-1)^k d_k \pmod{11}$. For example, $1234 \equiv 4 - 3 + 2 - 1 = 2 \pmod{11}$.]

3) [20 points] Prove that $x^2 + y^2 = 3{,}000{,}000{,}003$ has no solution with $x, y \in \mathbb{Z}$.

4) Congruences modulo 7: Part (b) is a HW problem. Part (a) [really simple] can help with part (b), but if you have a different solution that doesn't use (a), that is fine too.

(a) [10 points] Prove that $b \equiv 0 \pmod 7$ if and only if $-2b \equiv 0 \pmod 7$. [Remember that there are two parts to this: the "if" and the "only if".]

(b) [10 points] Prove that if the decimal digits of a are given by $a = d_k d_{k-1} \cdots d_1 d_0$, then a is divisible by 7 if and only if $d_k d_{k-1} \cdots d_1 - 2 \cdot d_0$ is divisible by 7. [For example, this means that 1234 is divisible by 7 if and only if $123 - 2 \cdot 4 = 115$ is divisible by 7.]

[**Hint:** You can use the previous part, even if you could not do it.]

5) [20 points] Prove that if $a, b, n \in \mathbb{Z}_{>0}$ and $a^n \mid b^n$, then $a \mid b$.

[**Hint:** This would have been much harder in the last exam.]