Midterm 3

Math 300 – Fall 2020

October 21st, 2020

Instructions

- *Write neatly and legibly!*
- Your camera *must* be on at *all times* and showing you properly. (You can only leave Zoom when you are done!)
- Leave the sound on (not the mic), so that you can *hear* incoming private messages or if I need to say something to all.
- You do not need to copy the statements. Just number your answers.
- Each problem must be solved in a different page, but items of the same problem can be in the same page.
- If you have any questions, send me a private message through the chat.
- You can only use your computer to look at the exam or to send me a message.
- **When you are done with the exam and are about to start scanning/uploading, send me a private message!** (Something like *"Scanning now."*)
- Make sure your scans are legible before uploading them to Canvas.
- **When you are done uploading your solutions, send me a private message.** (Something like *"Done."* No need for the time.) You can then leave Zoom.
- **Be prepared to, upon request (via private message), show me your surroundings!**

1) Let $\mathcal{F}$ and $\mathcal{G}$ be non-empty families of sets. Prove that if every element of $\mathcal{F}$ is a subset of *every* element of $\mathcal{G}$, then $\bigcup\mathcal{F} \subseteq \bigcap\mathcal{G}$.

2) Prove that for every integer n, n^3 is even if and only if n is even.
[**Note:** This was a HW problem.]

3) Let $\mathcal{F}$ and $\mathcal{G}$ be non-empty families of sets. Prove that $\bigcup(\mathcal{F} \cup \mathcal{G}) \subseteq (\bigcup\mathcal{F}) \cup (\bigcup\mathcal{G})$.

4) Let U be a set. Prove that there is a unique $A \in \mathscr{P}(U)$ such that for all $B \in \mathscr{P}(U)$ we have $A \cup B = A$.

5) Let A, B. C, and D be sets. Prove that if $A \times B$ and $C \times D$ are disjoint, then either A and C or B and D are disjoint.
[**Note:** This was a HW problem.]