Midterm (In Class)

M552 – Abstract Algebra

March 3rd, 2008

Solve as many as you can in class. You have one hour. Choose wisely the ones on which you will work first.
Of course, even if you did not do one part of a problem, you can use it for the later parts.

1. Modules:

(a) [10 points] Give an example of an *injective* homomorphism of [left] R-modules $\phi : N \to M$ such that $1 \otimes \phi : L \otimes_R N \to L \otimes_R M$ is *not* an injection for some [right] R-module L. [You do *not* have to repeat computations of tensor products that were done in class, in HW, or in Dummit and Foote.]

(b) [20 points] Let R be a commutative ring with 1. Show that if every R module is free, then R is a PID.

2. Linear algebra:

(a) [10 points] Let $B \in \mathrm{M}_n(\mathbb{C})$ [$n \times n$ matrices with entries in $\mathbb{C}$] be a *block diagonal* matrix. Prove that B is diagonalizable if, and only if, each block is. [You can use the algebra of block matrices without proof.]

(b) [20 points] Let $A, B \in \mathrm{M}_n(\mathbb{C})$ be two *diagonalizable* matrices. Prove that there is [a *single*] $P \in \mathrm{GL}_n(\mathbb{C})$ [i.e., an invertible matrix] such that *both* $P^{-1}AP$ and $P^{-1}BP$ are diagonal [so they are *simultaneously diagonalizable*] if, and only if, $AB = BA$. [**Hint:** Look at eigenspaces.]