Midterm (Take Home)

M552 – Abstract Algebra

March 3rd, 2008

- You are not supposed to discuss this with anyone.
- You can use Dummit and Foote and class notes, but please do not keep looking for solutions (in several books, papers, internet, etc.).
- Please, since you have some time, write your solutions neatly.
- The due date is Wednesday, 03/05 in class.
- If you feel you need more time, please let me know ASAP, so that all can have the same amount of time.

1. Let R be a *local ring*, i.e., a commutative ring with 1 with a unique maximal ideal, say I, and let M be a *finitely generated* R-module.

(a) [10 points] If N is a submodule of M and $M = N + (I \cdot M)$, then $M = N$.

[**Hint:** Last semester I proved *Nakayma's Lemma* for ideals. The same proof works for [finitely generated] modules. [See Proposition 16.1 on pg. 751 of Dummit and Foote.] Use it here.]

(b) [30 points] Suppose further that M is *projective* [still with the same hypothesis as above]. Prove that M is free.

[**Hints:** Look at $M/(I \cdot M)$ to find your candidate for a basis. Use (a) to prove it generates M. Then let F be a free module with the rank you are guessing to be the rank of M and use (a) to show that the natural map $\phi : F \to M$ is an isomorphism.]