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Math 251

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Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 75 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	15	
4	20	
5	25	
Total	100	

1) [20 points] Mark each statement as true (T) or false (F). [No need to justify your answers here.]

(a) For all $n \times n$ matrices A and B, we have $\det(A + B) = \det(A) + \det(B)$.

(b) For all $n \times n$ matrices A and B, we have $\det(A \cdot B) = \det(A) \cdot \det(B)$.

(c) If E is an elementary matrix, then $\det(E) = 1$.

(d) If A is a *singular* $n \times n$ matrix, then the system $A \cdot \mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(e) For all $m \times n$ matrices A and B and scalar k, we have $kB + kA = k(A + B)$.

(f) Every homogeneous system has a solution.

(g) Every system with more variables than equations has infinitely many solutions.

(h) For all $n \times n$ matrices A and scalar k, we have that $\det(kA) = k \det(A)$.

(i) For all $n \times n$ matrices A and B, we have $(A \cdot B)^{\mathrm{T}} = A^{\mathrm{T}} \cdot B^{\mathrm{T}}$.

(j) If A is a square matrix and $A\mathbf{x} = \mathbf{b}$ has a single solution, then so does $A^{\mathrm{T}}\mathbf{x} = \mathbf{b}$.

2) [20 points] *Quickies!* [These should take you 10 seconds each if you've studied.] You don't need to justify your answers.

(a) $\begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix}^{-1} =$

(b) $\begin{vmatrix} -3 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 0 & 0 \\ -2 & 2 & 4 & 3 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{vmatrix} =$

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} =$

(d) If $E \cdot \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 0 \\ 5 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 5 & -4 & 1 \\ 0 & 2 & 0 \end{bmatrix}$, then $E =$

(e) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$, then $\begin{vmatrix} 2g & 2h & 2i \\ d+3a & e+3b & f+3c \\ -a & -b & -c \end{vmatrix} =$

3) [15 points] Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \\ 0 & 5 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}.$$

Compute $A^{\mathrm{T}} \cdot B \cdot \mathrm{tr}(C)$.

4) [20 points] Let

$$A = \begin{bmatrix} 4 & 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 & -3 \\ 2 & 1 & 2 & 3 & 0 \\ 1 & 2 & 0 & 2 & 6 \end{bmatrix}.$$

How many solutions does the system $A\mathbf{x} = \mathbf{0}$ have? [I'm not asking you to solve it, just for the number of solutions.]

5) [25 points] Let

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Find $\det(A^3)$, A^{-1} [if possible; if not, justify], and solve $A\mathbf{x} = \mathbf{b}$. [Don't work too hard here! Think a little before you start!]

Scratch: