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Math 251

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Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 75 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 7 questions and 10 printed pages (including this one, a page for scratch work in the end, and a last page with the definition of vector spaces).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	20	
3	15	
4	15	
5	15	
6	10	
7	15	
Total	100	

1) [10 points] Let $\mathbf{u} = (1, 1)$ and $\mathbf{v} = (3, k)$. Find k such that the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\pi/4$.

2) [20 points] Give the matrix that represents the following linear transformation (from $\mathbb{R}^3$ to $\mathbb{R}^3$): projection to the xy-plane, followed by a rotation of $\pi/2$ around the z-axis, followed by a reflection on the yz-plane.

3) [15 points] Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(2,1) = (3,0)$ and $T(0,1) = (1,-2)$. Find the inverse of T or show that such inverse does not exist.

4) [15 points] Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation and assume that 0 is an eigenvalue of T. Can T be one-to-one? Can it be onto? Justify your answers!

5) [15 points] Let $V = \mathbb{R}^2$ with the usual sum of vectors in $\mathbb{R}^2$, but with the following multiplication by real numbers: $k(x, y) = (ky, kx)$. Show that V is *not* a vector space.

6) [10 points] Consider the set of all matrices

$$\begin{bmatrix} a & 1 \\ 2 & b \end{bmatrix}, \qquad a, b \in \mathbb{R}.$$

with:

$$\begin{bmatrix} a & 1 \\ 2 & b \end{bmatrix} + \begin{bmatrix} a' & 1 \\ 2 & b' \end{bmatrix} = \begin{bmatrix} a + a' & 1 \\ 2 & b + b' \end{bmatrix}, \qquad k \begin{bmatrix} a & 1 \\ 2 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 2 & kb \end{bmatrix}.$$

This set with this sum and scalar multiplication *is* a vector space. [You do not need to prove it! Just take my word for it.] What is the zero of this vector space? What is

$$-\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}?$$

7) [15 points] Let V be the set of all functions [from $\mathbb{R}^2$ to $\mathbb{R}^2$] of the form $f(x,y) = (ax + by, ax^2 + by^2)$, where $a, b \in \mathbb{R}$. Is V a vector space with the usual sum and scalar multiplication of functions? [Justify!]

Scratch:

Vector Space Requirements

A non-empty set V with a sum and a scalar product is a vector space if it satisfies the following conditions:

0. $\mathbf{u}+\mathbf{v} \in V$ for all $\mathbf{u}, \mathbf{v} \in V$, and $k\mathbf{u} \in V$ for all $\mathbf{u} \in V$ and $k \in \mathbb{R}$;
1. $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in V$;
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$;
3. there is $\mathbf{0} \in V$ such that $\mathbf{0}+\mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in V$;
4. given $\mathbf{u} \in V$, there exists $-\mathbf{u} \in V$ such that $\mathbf{u}+(-\mathbf{u}) = \mathbf{0}$;
5. $k(\mathbf{u}+\mathbf{v}) = k\mathbf{u}+k\mathbf{v}$ for all $\mathbf{u}, \mathbf{v} \in V$ and $k \in \mathbb{R}$;
6. $(k+l)\mathbf{u} = k\mathbf{u}+l\mathbf{u}$ for all $\mathbf{u} \in V$ and $k, l \in \mathbb{R}$;
7. $k(l\mathbf{u}) = (kl)\mathbf{u}$ for all $\mathbf{u} \in V$ and $k, l \in \mathbb{R}$;
8. $1\mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in V$.