February 21st, 2011

Math 141

Luís Finotti
Spring 2011

Name: ....

Student ID (last 6 digits): XXX- ....

TA recitation (check one):

Cody Lorton: ☐ 8:00 ☐ 9:05   Craig Collins: ☐ 11:15 ☐ 12:25

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	30	
2	15	
3	20	
4	15	
5	20	
Total	100	

1) Compute the following limits. If they do not exist or are infinite, check if the side limits exist.

(a) [5 points] $\lim_{x \to -\infty} \dfrac{3x^3 - x + 1}{x^3 + x^2}$

(b) [5 points] $\lim_{x \to 2} \dfrac{\dfrac{1}{x-1} - 1}{x - 2}$

(c) [10 points] $\lim_{x \to 2} \dfrac{x^2 + x - 6}{(x-2)^2}$

(d) [10 points] $\lim_{x \to -\infty} \sqrt{x^2 - 3x + 1} - 2x$

2) [15 points] Compute the following derivative (using the formulas, no need for limits).

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x \cdot 2^x - 2x\sqrt{x}}{3x^4 - x^2 + 1}\right)$$

No need to simplify! Note that you might get less partial credit if you skip steps and get the wrong answer. [No penalty if the answer is correct.]

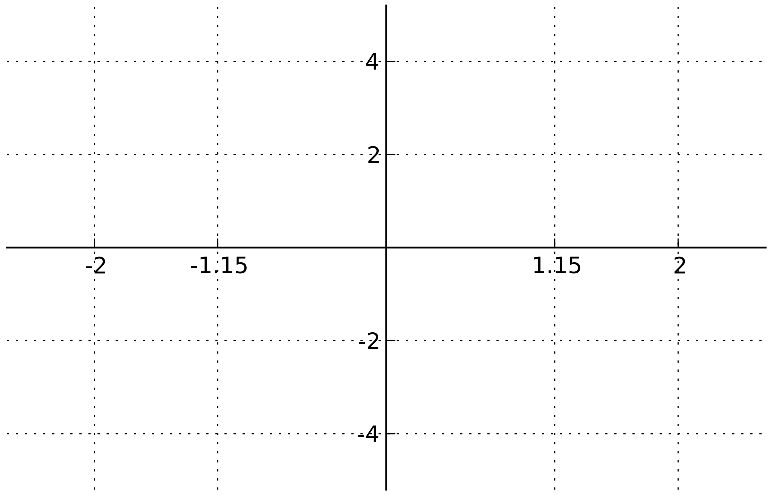
3) [20 points] Let

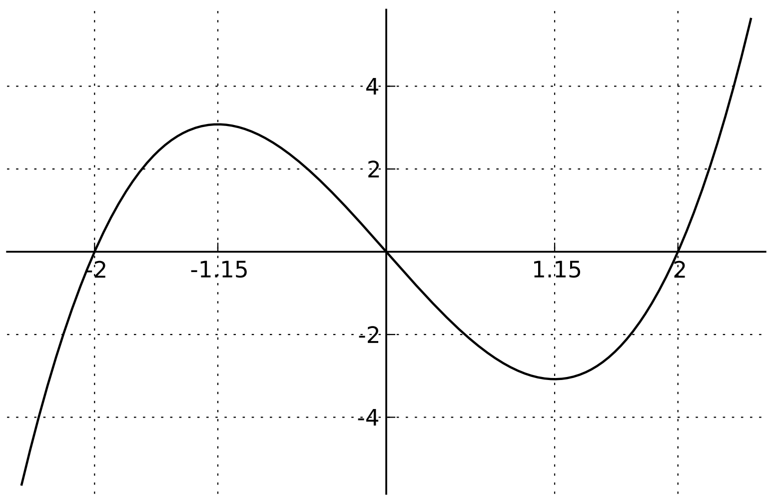
$$f(x) = \begin{cases} x^3, & \text{if } x < 0, \\ x^2, & \text{if } x \geq 0. \end{cases}$$

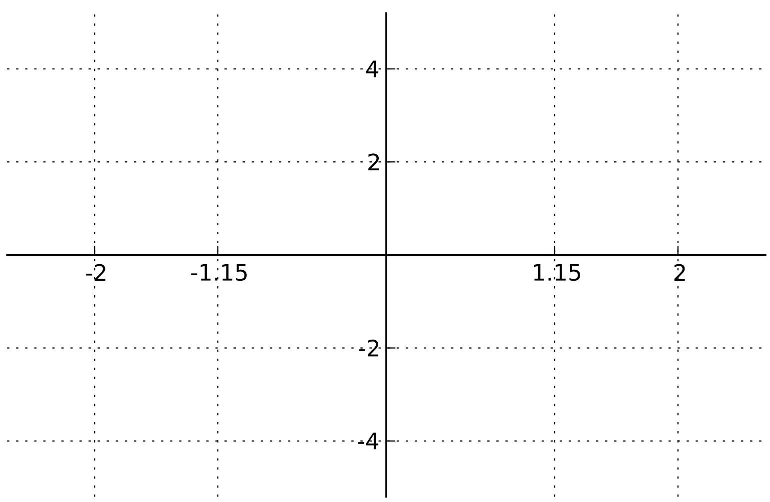
Is $f(x)$ continuous at $x = 0$? Is it differentiable? If so, compute $f'(0)$. **Note:** You *have* to use limits for differentiability.

4) [15 points] Let $f(x) = x\mathrm{e}^x - x^6 + 1$. Show that $f(x)$ has at least two zeros [i.e., there are $a, b \in \mathbb{R}$, with $a \neq b$, such that $f(a) = f(b) = 0$].
Hint: I am not asking you to *find* these zeros, just show their existence. Also, e is approximately 2.72.

5) [20 points] The graph in the middle is the graph of $y = f'(x)$ for some function f such that $f(0) = 0$. Sketch $y = f(x)$ in the grid *above* the given graph and beneath it sketch the graph of $y = f''(x)$.







Scratch: