April 6th, 2016

Math 351

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Spring 2016

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your index cards.

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) Congruences:

(a) [10 points] Find all $x \in \mathbb{Z}$ such that

$$4x \equiv 10 \pmod{30}.$$

If there is no such x, simply justify why.

(b) [10 points] Find all $x \in \mathbb{Z}$ satisfying [simultaneously]:

$$\begin{aligned} x &\equiv 2 \pmod{7}, \\ x &\equiv 3 \pmod{11}. \end{aligned}$$

If there is no such x, simply justify why.

2) [20 points] Let R be a *non-commutative* ring and $a \in R$ such that there are $b, c \in R$ such that $ba = 1$ and $ac = 1$. Prove that $b = c$. *Justify each step!*

3) Examples:

(a) [10 points] Give an example of an *infinite field* F such that $6 \cdot a = 0$ for all $a \in F$. [**Hint:** Can you find a finite example first?]

(b) [10 points] Give an example of a ring R that contains $\mathbb{C}[x]$ as a *proper* subring [i.e., $\mathbb{C}[x] \subseteq R$, $\mathbb{C}[x]$ a subring of R, but $\mathbb{C}[x] \neq R$].

4) [20 points] Prove that

$$R = \{f \in \mathbb{Z}[x] \ : \ f = a + x^2 f_1 \text{ for some } a \in \mathbb{Z} \text{ and } f_1 \in \mathbb{Z}[x]\}$$

is a domain.

5) Let R be a commutative ring [but not necessarily a domain] and let $f, g \in R[x] \setminus \{0\}$, with $\deg(f) = \deg(g)$ and $f \mid g$.

(a) [10 points] Prove that if R is a domain, then there is $a \in R$ such that $g = a \cdot f$.

(b) [10 points] Prove that in $\mathbb{I}_6[x]$, if $f = \bar{2}x + \bar{1}$ and $g = \bar{5}x + \bar{1}$, then $f \mid g$. [So, the statement in (a) does not hold for non-domains, as clearly $g \neq a \cdot f$ for any $a \in \mathbb{I}_6$].

Scratch: