May 6th, 2016

Math 351

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Spring 2016

Name:..

Student ID (last 6 digits): XXX- ....

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 7 questions and 12 printed pages (including this one and two pages for scratch work in the end).

No books or notes are allowed on this exam!

Questions 3, 4 and 5 will make up your Midterm 2 grade.

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	25	
7	25	
Total	100	

1) Remainders:

(a) [5 points] Find the remainder of 2^{87} when divided by 7.

(b) [5 points] Find the remainder of $47300272^{63745765}$ when divided by 3.

2) [10 points] Let $a, b, c \in \mathbb{Z} \setminus \{0\}$ and $d = \gcd(a, b)$. Prove that $\gcd(a, b, c) = \gcd(d, c)$.
[**Hint:** Prove first that n is a common divisor of a, b and c iff it is a common divisor of d and c.]

3) [10 points] Find all $x \in \mathbb{Z}$ satisfying [simultaneously]:

$$\begin{aligned} 3x &\equiv 1 \pmod{7}, \\ x &\equiv 4 \pmod{11}. \end{aligned}$$

If there is no such x, simply justify why.

4) Polynomials:

(a) [5 Points] Give an example of a degree one polynomial [with coefficients in some ring you have to choose] with two distinct roots.

(b) [5 points] Let F be a *field*, $f, g \in F[x] \setminus \{0\}$, with $\deg(f) = m$, $\deg(g) = n$ and $m \geq n$. Show that if there are $(m+1)$ *distinct* elements in F, say $\{a_1, a_2, \ldots, a_{m+1}\}$, such that $f(a_i) = g(a_i)$ for $i = 1, \ldots, m+1$, then $f = g$.

[**Hint:** Look at the polynomial $f - g$. To show $f = g$, suffices to show that $f - g = 0$.]

5) Examples:

(a) [5 points] Give an example of an *infinite field* F such that $6 \cdot a = 0$ for all $a \in F$. [**Hint:** Can you find a finite example first?]

(b) [5 points] Give an example of a ring R that contains $\mathbb{C}[x]$ as a *proper* subring [i.e., $\mathbb{C}[x] \subseteq R$, $\mathbb{C}[x]$ a subring of R, but $\mathbb{C}[x] \neq R$].

6) Determine if the polynomials below are irreducible or not in the corresponding polynomial ring. *Justify each answer!*

(a) [4 points] $f = x^2 - \sqrt{7}x + 2$ in $\mathbb{R}[x]$.

(b) [4 points] $f = x^7 + \mathrm{e}\, x^5 - \pi\, x^2 + \sqrt{5}\, x + \log(2)$ in $\mathbb{C}[x]$.

(c) [4 points] $f = \overline{211}\, x - \overline{301}$ in $\mathbb{F}_{521}[x]$.

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(d) [4 points] $f = x^7 + 4x^6 - 8x^4 + 120x^3 - 2x + 14$ in $\mathbb{Q}[x]$.

(e) [5 points] $f = 4x^3 + 3x^2 - 34x + 3001$ in $\mathbb{Q}[x]$.

(f) [4 points] $f = x^6 - 2x^5 + x^4 - 3x^2 + x + 2$ in $\mathbb{Q}[x]$.

7) Let $\sigma, \tau \in S_9$ be given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 5 & 4 & 3 & 9 & 2 & 8 & 6 \end{pmatrix} \quad \text{and} \quad \tau = (1\,3\,8)(2\,4\,5\,9).$$

(a) [5 points] Write the *complete* factorization of σ into disjoint cycles.

(b) [4 points] Compute σ^{-1}. [Your answer can be in any form.]

(c) [4 points] Compute $\tau\sigma$. [Your answer can be in any form.]

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(d) [4 points] Compute $\sigma\tau\sigma^{-1}$. [Your answer can be in any form.]

(e) [4 points] Write τ as a product of transpositions.

(f) [4 points] Compute $\text{sign}(\tau)$.

Scratch:

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