Errata 1

Math 652

February 4, 2016

I made a mistake in class when hurrying to finish the decomposition of finite commutative F-algebras in class today. I've stated that if $\{e_1, \ldots, e_r\}$ is the set of *all* primitive idempotents of A, then $A \cong \prod_{i=1}^{r} Ae_i$ when trying to prove the uniqueness (up to ordering) of a decomposition of an algebra into indecomposables. This is true, but I hadn't proved it yet. [I was then assuming part of what I wanted to prove.]

Here is the proper proof of the uniqueness:

Theorem. *If $A \cong \prod_{i=1}^{r} A_i$, with A_i indecomposable, and if $\{e_1, \ldots, e_r\}$ is the system of orthogonal idempotents associated to this representation, then these are* all *the primitive idempotents of A. Since $A_i \cong A \cdot e_i$, all decompositions of A into indecomposable must be the same [namely, $A \cong \prod_{i=1}^{r} A \cdot e_i$] up to the order of and isomorphism of the factors.*

Proof. Remember we had proved that the e_i's are primitive [since the A_i's are indecomposable]. Also, we have that $\sum_{i=1}^{r} e_i = 1$ [since they are the system of orthogonal idempotents associated to a representation as direct product]. If there is some e primitive idempotent different from all e_i's, then [by a proposition in class] $e \cdot e_i = 0$. But then,

$$e = e \cdot 1 = e \cdot \sum_{i=1}^{r} e_i = \sum_{i=1}^{r} e \cdot e_i = \sum_{i=1}^{r} 0 = 0,$$

a contradiction. So, $\{e_1, \ldots, e_r\}$ is the complete set. $\square$