April 17th, 2018

Math 241

Luís Finotti
Spring 2018

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 4

Version A

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 10 printed pages (including this one and a page for scratch work in the end).

No calculators, books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	15	
3	15	
4	25	
5	25	
Total	100	

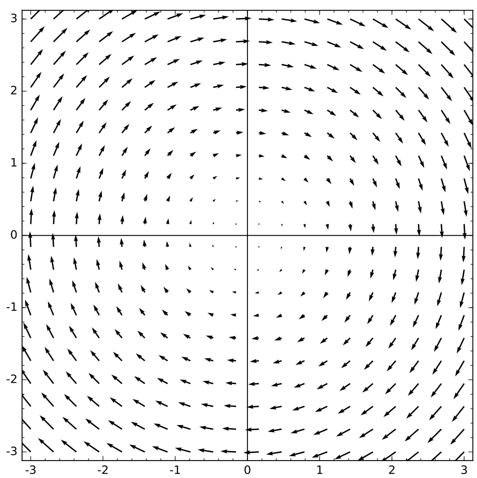
1) Let $\mathcal{D}$ be the region on the plane given by $4 \leq x^2 + y^2 \leq 9$ and $y \geq x$.

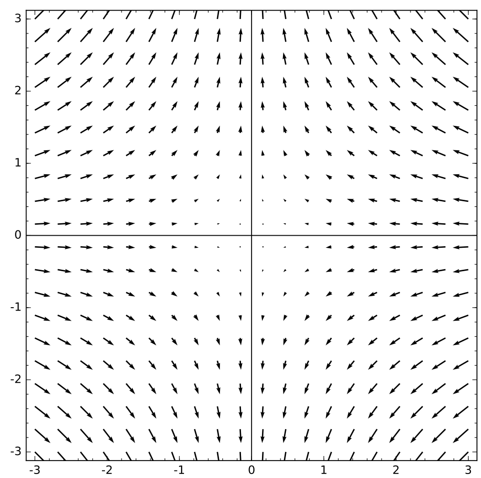
(a) [10 points] Sketch $\mathcal{D}$.

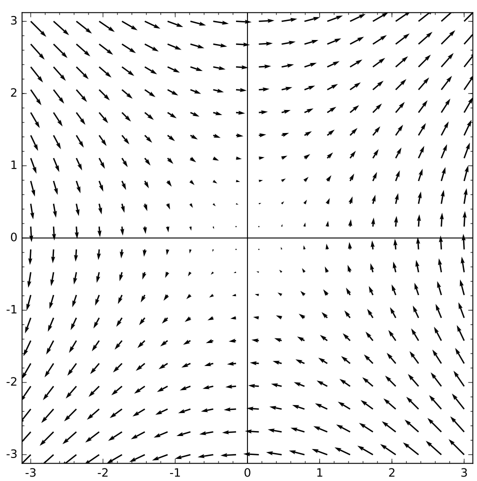
(b) [10 points] Use *polar coordinates* to set up the integral $\displaystyle\iint_{\mathcal{D}} x - y \, \mathrm{d}A$. [You do not need to compute it, just set up as iterated integrals.]

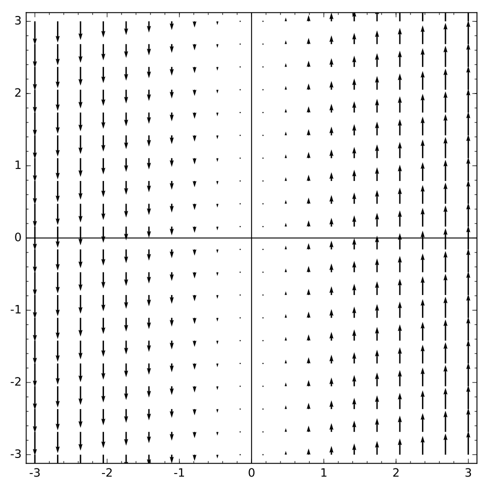
2) [15 points] Match the vector fields to their corresponding plots by writing the corresponding letters by the square containing the plot. (Leave the plots with no match unmarked.)

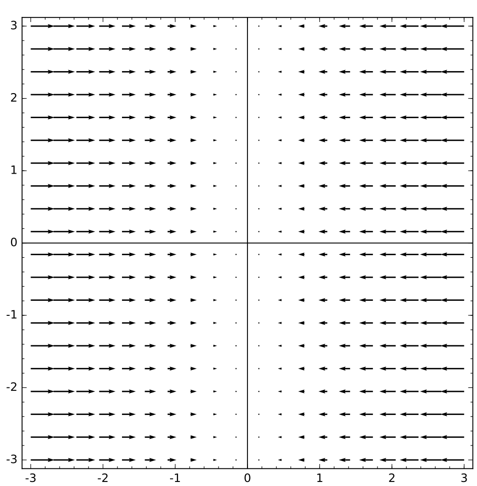
A: $\mathbf{F}(x, y) = \langle 0, y \rangle$ B: $\mathbf{F}(x, y) = \langle -x, y \rangle$

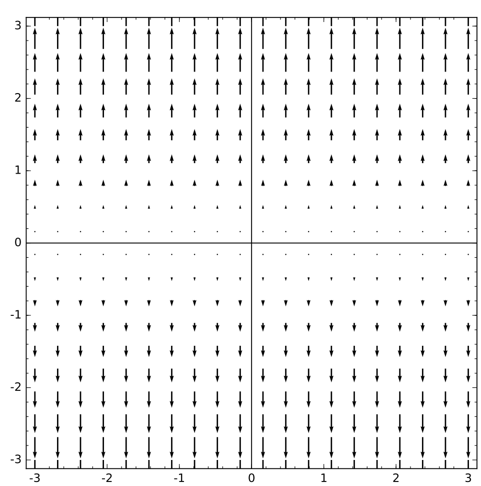




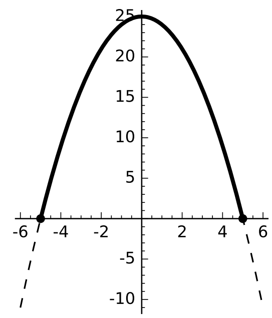








3) [15 points] Let $\mathbf{F}(x, y) = \langle x^2 y, y^2 \rangle$ and $\mathcal{C}$ be the piece of the parabola $y = 25 - x^2$ going from $(-5, 0)$ to $(5, 0)$. [See picture below.] Set up [but do not compute!] the vector line integral $\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ as a simple Calculus 2 integral.



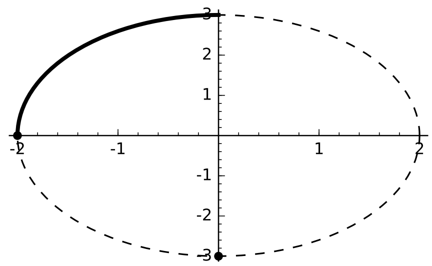
4) Let $\mathbf{F} = \langle g, h \rangle = \langle 2x + y^2 - \mathrm{e}^{x+3y}, 1 + 2xy - 3\mathrm{e}^{x+3y} \rangle$.

(a) [8 points] Show that $\mathbf{F}$ is a conservative vector field *without computing its potential.*

(b) [9 points] Find the potential of $\mathbf{F}$.

Continues on next page!

(c) [8 points] Let $\mathcal{C}$ be the part of ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ between $(-2, 0)$ and $(0, 3)$, going *clockwise*. [See picture below.] Compute $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$. [Here it is not enough to simply set it up!]



5) Let $\mathcal{D}$ be the region given by $x \leq y \leq 2x$ and $3 \leq x + y \leq 6$ and let $G(u, v) = \left(\frac{u}{v+1}, \frac{uv}{v+1}\right)$.

(a) [6 points] Sketch the region $\mathcal{D}$.

(b) [6 points] Compute $\text{Jac}(G)$, the Jacobian determinant of $G(u, v)$.

Continues on next page!

(c) [7 points] Sketch the domain $\mathcal{D}_0$ such that $G(\mathcal{D}_0) = \mathcal{D}$ in the (u, v)-plane.

[**Note:** $\mathcal{D}_0$ is simply the new region of integration when you make the change of variables using $G(u, v)$ above.]

Continues on next page!

(d) [6 points] Set up [but do not compute] the integral $\iint_{\mathcal{D}} x/y\,\mathrm{d}A$ *by changing the variables* using $G(u,v)$ above [i.e., $G(u,v) = \left(\frac{u}{v+1}, \frac{uv}{v+1}\right)$].

[**Note:** If you could not do part (c) to find the new domain of integration, use $\mathcal{D}_0 = [1,2] \times [2,3]$ instead.]

Scratch: