October 13th, 2017

Math 351

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Fall 2017

Name: . . . .

Student ID (last 6 digits): XXX- . . . .

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No calculators, books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Compute the remainder of 2^{2839} when divided by 13. [Show work!]

2) [20 points] Find all integers x such that

$$\begin{aligned} 5x &\equiv 7 \pmod{8} \\ 2x &\equiv 4 \pmod{10}. \end{aligned}$$

[If there is no such integer, explain how you could tell.]

3) [20 points] Prove that there are no positive integers a and b such that

$$\gcd(a, b) = 2^5 \cdot 3^4 \cdot 7 \cdot 11^2,$$
$$\operatorname{lcm}(a, b) = 2^8 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 11^2.$$

[Make it *very* clear what results you are using!]

4) [20 points] Show that if x, y and z are integers such that $x^4 + y^4 = z^4$, then at least one of them is divisible by 3.

5) [20 points] Let $a \in \mathbb{Z}_{\geq 2}$ with prime factorization

$$a = p_1^{e_1} \cdot p_2^{e_2} \cdots p_k^{e_k}$$

[p_i's distinct primes and $e_i \in \mathbb{Z}_{>0}$]. Prove that a is a perfect square [i.e., $a = b^2$ for some $b \in \mathbb{Z}$] if and only if e_i is even for all i.
[**Note:** This was a HW Problem.]

Scratch: