February 5th, 2020

Math 351

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Spring 2020

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] Use the *Extended Euclidean Algorithm* to write the GCD of 117 and 66 as a linear combination of themselves. *Show work!*

[**Hint:** You should get 3 for the GCD!]

2) [20 points] Express 2020 in base 5, i.e., write

$$2020 = \boxed{?} + \boxed{?} \cdot 5 + \boxed{?} \cdot 5^2 + \boxed{?} \cdot 5^3 + \cdots$$

with the blanks in $\{0, 1, 2, 3, 4\}$. *Show work!*

[**Note:** Trial and error is not acceptable here! You have to use some algorithm that always works, like the one I showed you in class.]

3) [20 points] Prove that $\sqrt{6}$ is not a rational number, i.e., that $\sqrt{6}$ is not of the form a/b, with $a, b \in \mathbb{Z}_{>0}$.

[**Hint:** We've proved in class that $\sqrt{2}$ is not rational. This is *very* similar.]

4) [20 points] Let $n \in \mathbb{Z}$ and suppose that $3 \nmid n$. Prove that $\gcd(n, n+3) = 1$.

[**Note:** In the HW you've proved that for all $n \in \mathbb{Z}$, we have $\gcd(n, n+1) = 1$. This is similar.]

5) [20 points] Let a, b, and n be integers. [You may assume they are non-zero, if you wish.] Prove that if $a \mid n$, $b \mid n$, and $\gcd(a, b) = 1$, then $ab \mid n$.

[**Hint:** This was a HW problem.]

Scratch: