March 6th, 2020

Math 351

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Spring 2020

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) [25 points] Compute the remainder of 2^{5353} when divided by 11. [Show work, including computations!]

2) [25 points] Find all integers x such that

$$\begin{aligned} 3x &\equiv 7 \pmod{10} \\ 2x &\equiv 4 \pmod{14}. \end{aligned}$$

[If there is no such integer, explain how you could tell. *You need to show work!* Guessing solutions doesn't yield *any* credit.]

3) [25 points] Prove that there are no integers x, y, z such that $x^2 + y^2 + z^2 = 999$.

[**Note:** This was a HW problem. *You need to show work!*]

4) [25 points] Prove that if $a, b \in \mathbb{Z}_{\geq 2}$ are such that both $\gcd(a, b)$ and $\operatorname{lcm}(a, b)$ are *squares*, then both a and b must also be squares.

[**Hint:** In your HW you've proved that if $c \in \mathbb{Z}_{\geq 2}$ and its factorization into primes is $c = p_1^{g_1} \cdots p_k^{g_k}$, then c is a square if and only if all g_i's are *even*. You can use this here without proving it.]

Scratch: